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Tries

Data Structures and Algorithms for CL III, WS 2019-2020

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Data Structures & Algorithms in Python

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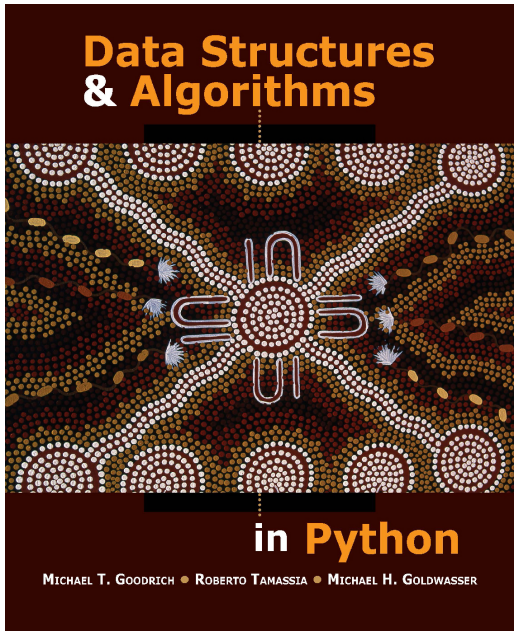


13.5 Tries

- Standard Tries
- Compressed Tries
- Suffix Tries

Standard Tries

Standard Tries

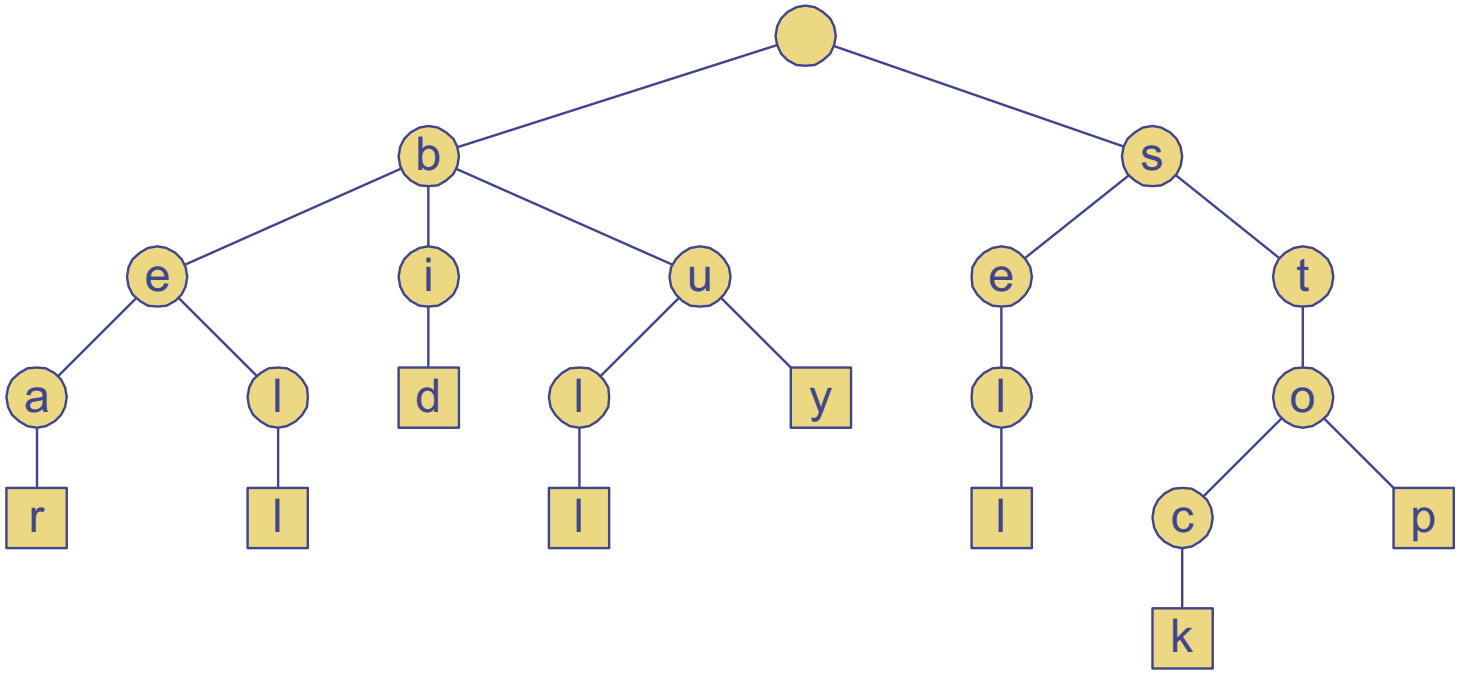
- a trie (pronounced „try“) is a tree-based data structure for storing strings in order to support fast pattern matching
- Main application: information retrieval
- Primary query operations supported by tries: pattern matching, prefix matching
- Approach suitable for applications where a series of queries is performed on a fixed text, such that the initial cost of preprocessing the text is compensated by a speedup in each subsequent query
- Example:
  - Website that offers pattern matching in works by Shakespeare
  - Text is large, immutable and often searched for
- Trie: compact data structure for representing a set of strings, e.g. all the words in a text
  - Supports pattern-matching queries in time proportional to the pattern size

Standard Tries – Formal Definition

- Let S be a set of s strings from alphabet Σ such that no string in S is a prefix for another string
- A standard trie for the set of strings S is an ordered tree T such that:
  - Each node of T, except the root, is labeled with a character from Σ
  - The children of an internal node of T have distinct labels and are alphabetically ordered
  - T has s leaves, each associated with a string of S, such that the concatenation of the labels of the nodes on the path from the root to a leaf v of T yields the string of S associated with v

Standard Tries - Example

- Standard trie for the set of strings $S = \{bear, bell, bid, bull, buy, sell, stock, stop\}$



Standard Tries - Properties

- An internal node can have anywhere between 1 and |Σ| children
  - In practice the average degree of internal nodes is small
  - On larger datasets, the average degree of nodes decreases with the depth of the tree (fewer strings sharing a common prefix)
  - In many languages there are character combinations that are unlikely to occur
- There is an edge connecting the root node to a child node for every character from Σ that is the first character of a string from S
- A path connecting the root node to an internal node v at depth k corresponds to a k-character prefix $X[0:k]$ of a string X of S
  - A trie stores the common prefixes in a set of strings

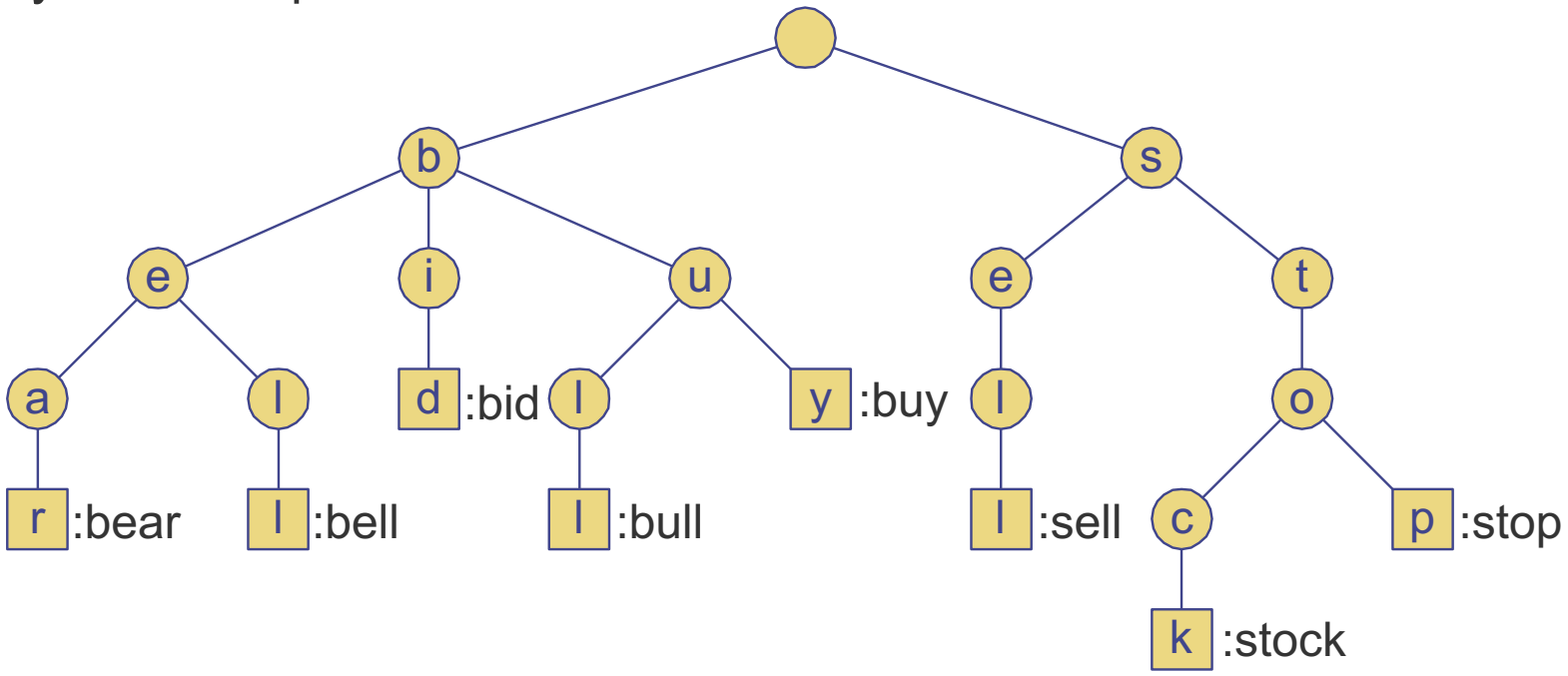
Standard Tries – Properties (cont'd)

- The following properties hold for a standard trie T storing a collection S containing s strings of total length n from an alphabet Σ:
  - The height of the trie T is equal to the length of the longest string in S
  - Every internal node of T has at most $|\Sigma|$ children
  - T has s leaves
  - The number of nodes of T is at most $n + 1$
    - Worst case: no two strings share a common, non-empty prefix – i.e. except for the root, all internal nodes have only one child

Trie Application: Map with String Keys

- A search in a trie T for the string X can be performed by tracing down from the root the path indicated by the characters of X
  - If the path can be traced and terminates in a leaf node - X is a key in the map
  - If the path cannot be traced, or it can be traced but terminates at an internal node – X is not a key in the map

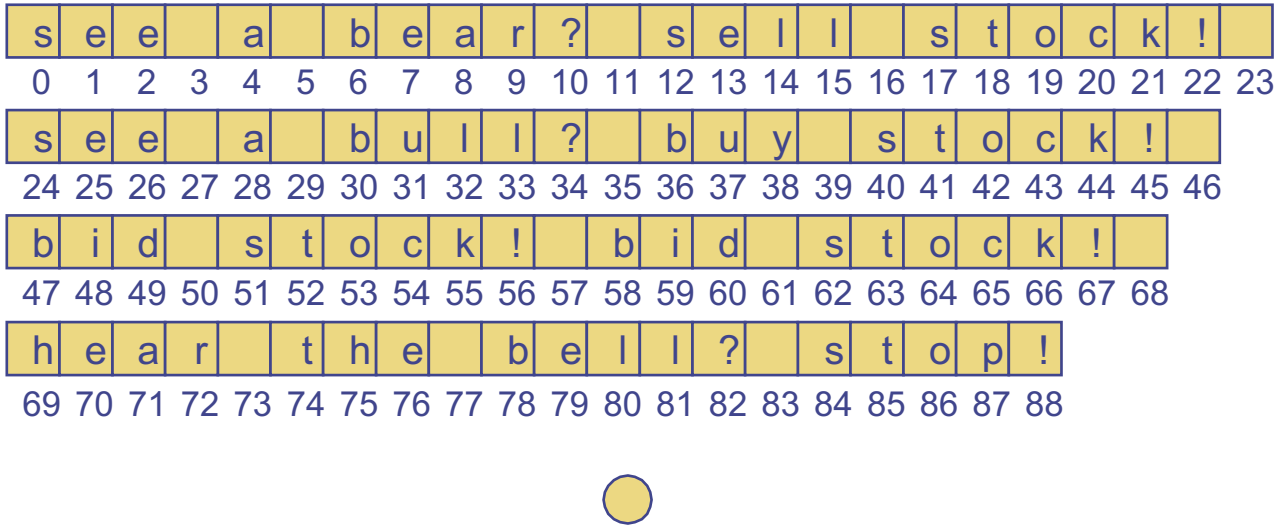
- bear
- big
- be



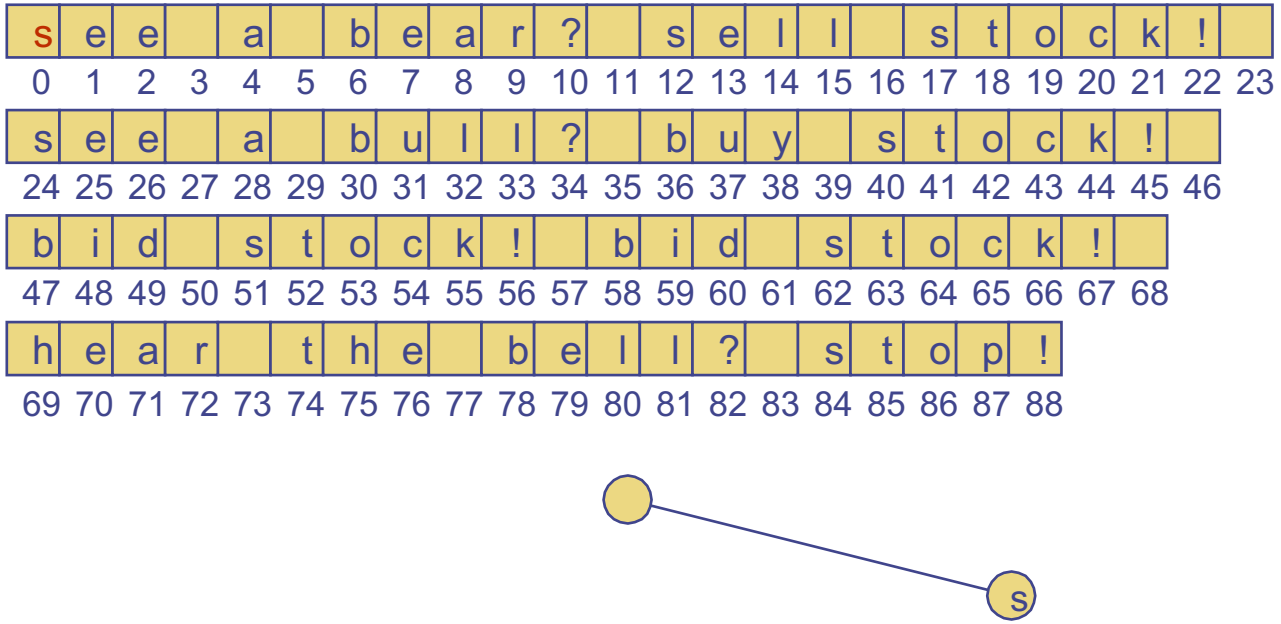
Trie Application: Map with String Keys (cont'd)

- Running time for searching for a string X of length m
  - At most $m + 1$ nodes of T are visited (the root + each of the characters)
    - At each node we spend at most $O(|\Sigma|)$ time determining what edge to follow next – that is – finding the child node which has the next character as its label
    - $O(|\Sigma|)$ is achievable even if the children are unordered – each node has at most $|\Sigma|$ children
  - Time can be improved by mapping characters to children by using at each node:
    - a secondary search table - $O(\log|\Sigma|)$
    - a hash table - $O(1)$
    - a direct lookup table of size $|\Sigma|$, if $|\Sigma|$ is small enough - $O(1)$
  - Typically, the search for a string of length m runs in $O(m)$ time

Word Matching with a Trie



Word Matching with a Trie (cont'd)



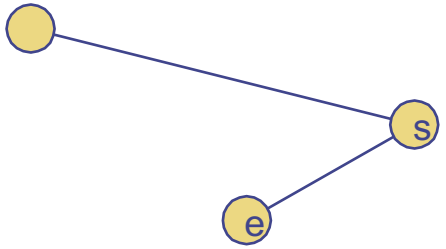
Word Matching with a Trie (cont'd)

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

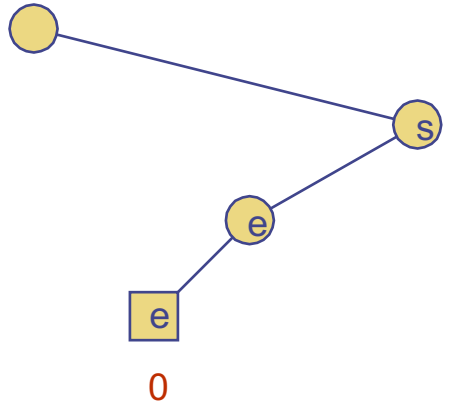
s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

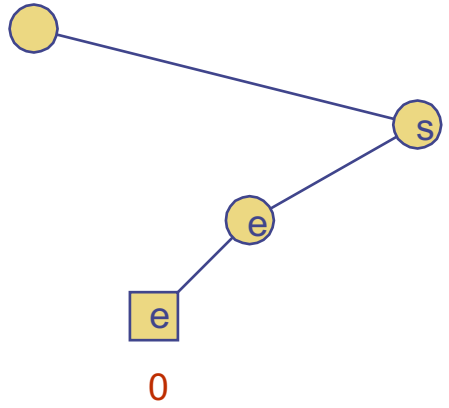
h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88



Word Matching with a Trie (cont'd)



Word Matching with a Trie (cont'd)



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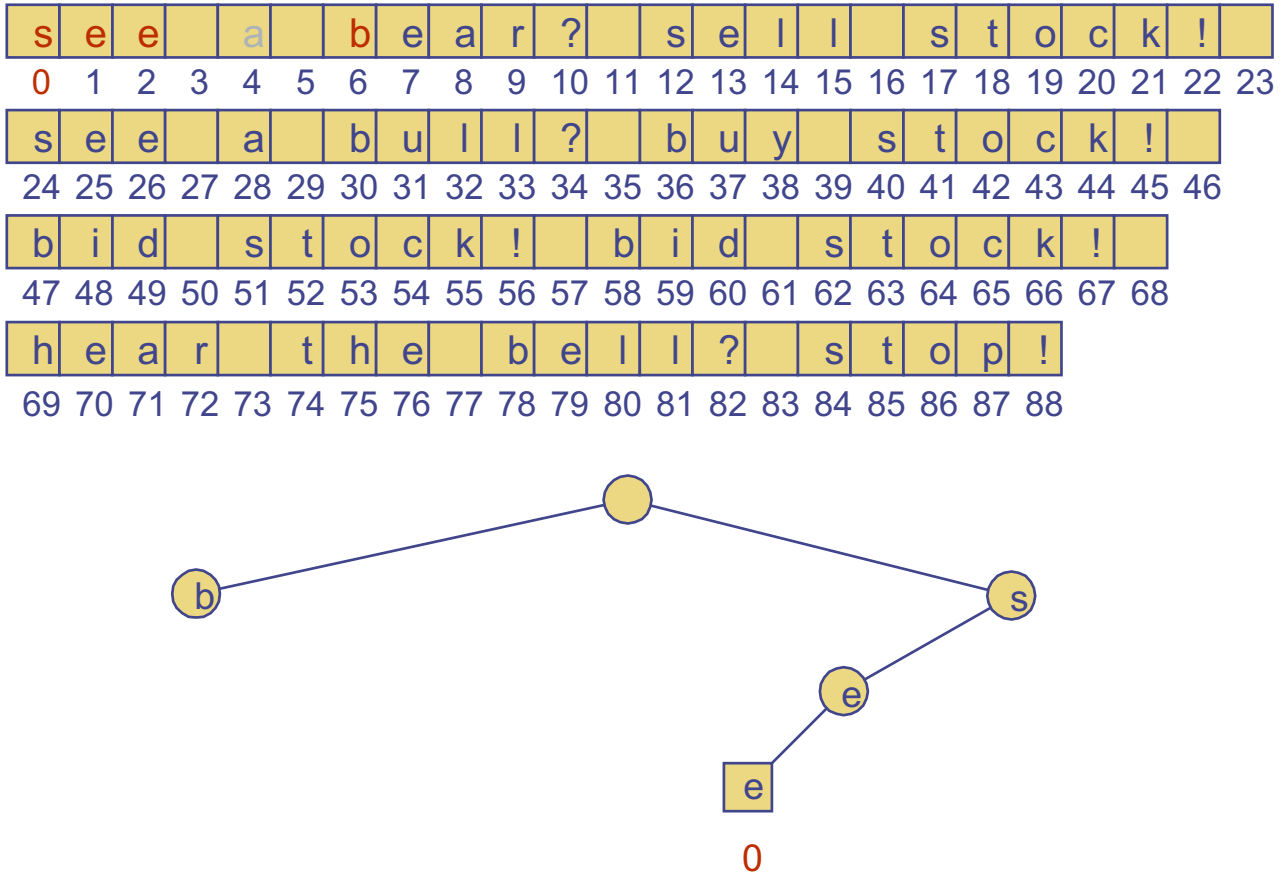
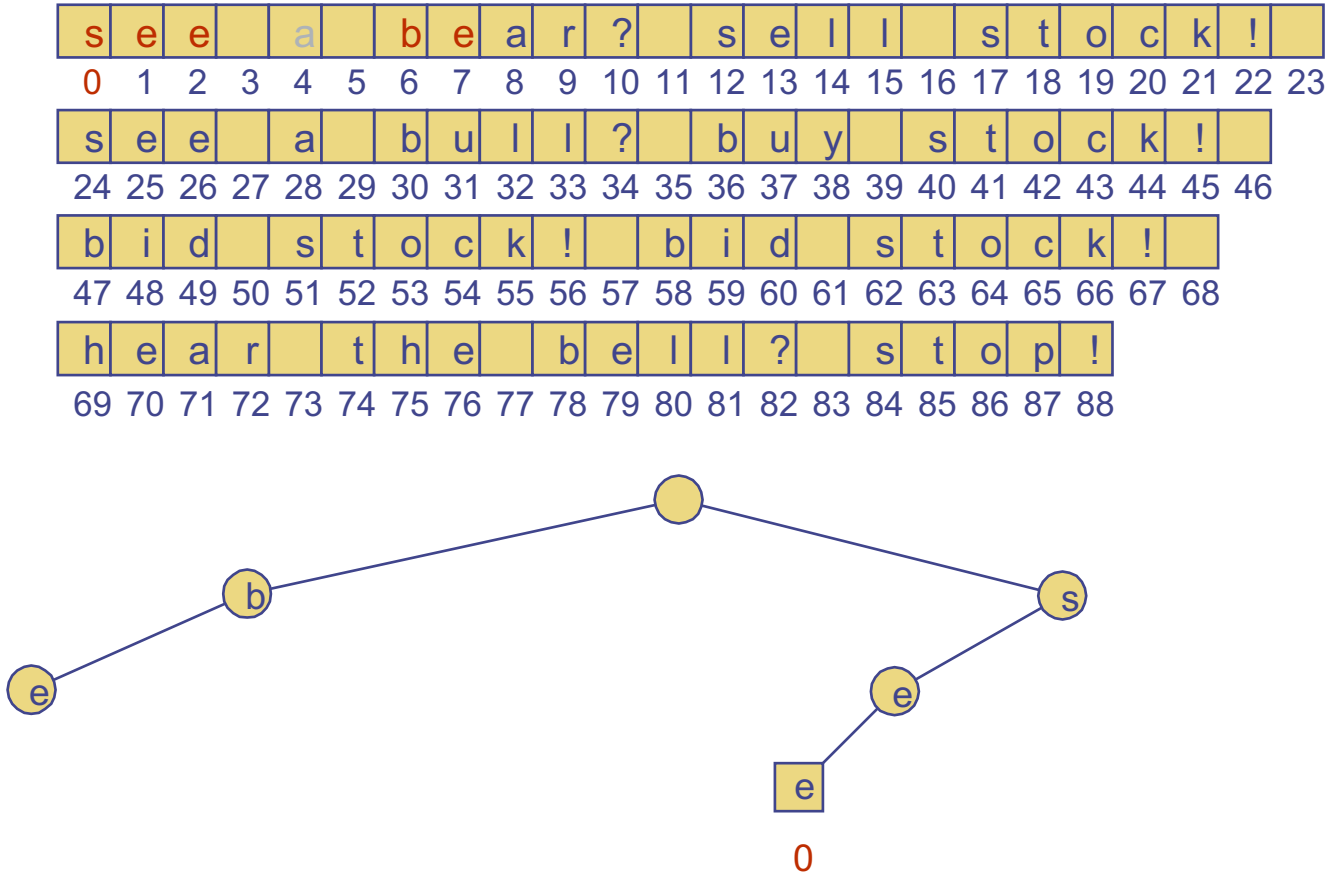
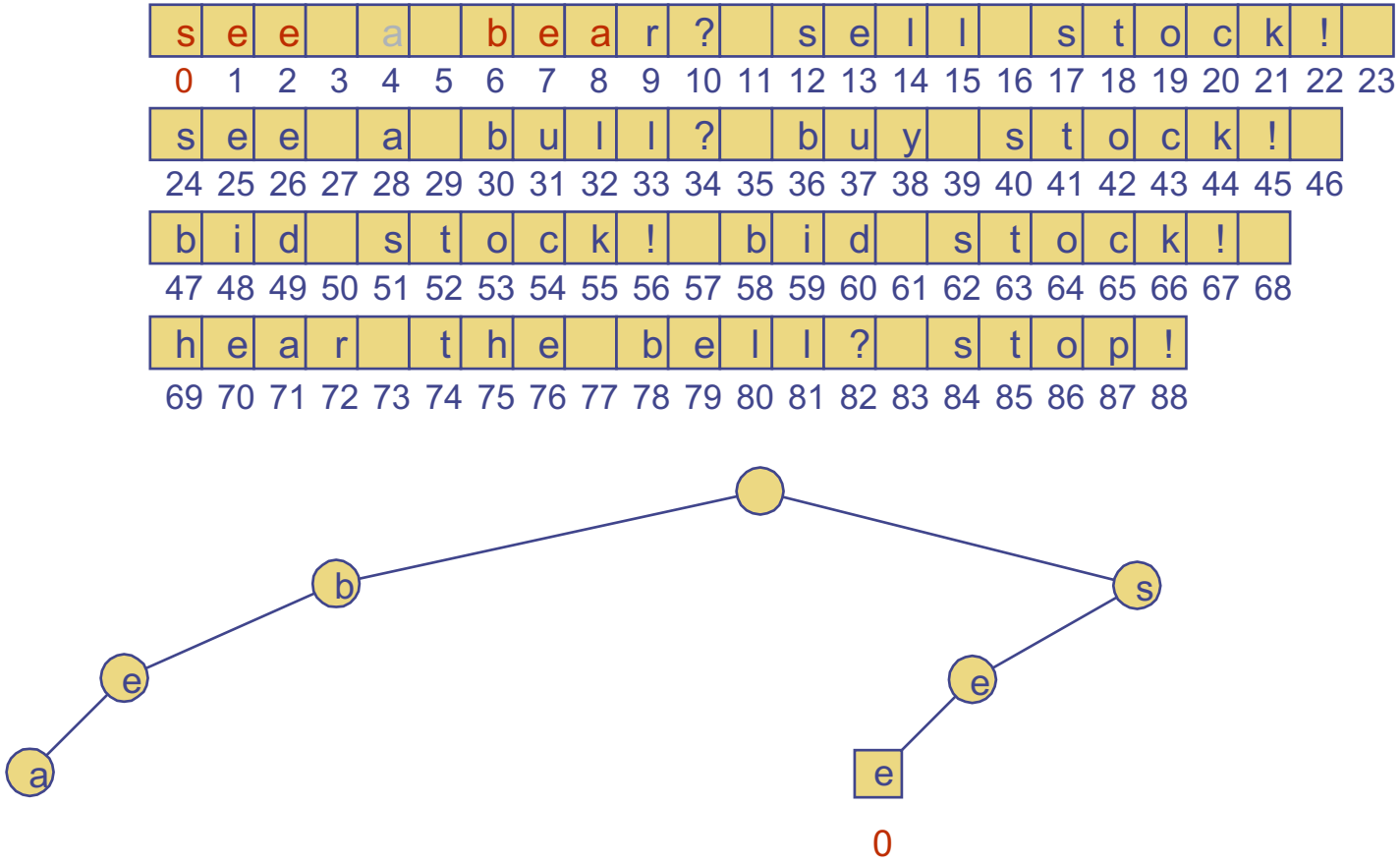


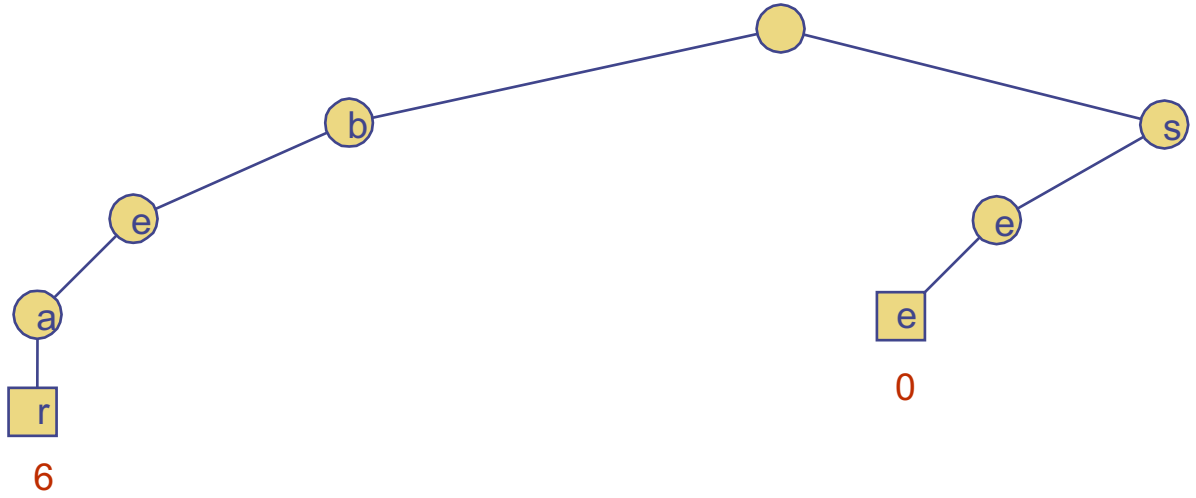
Word Matching with a Trie (cont'd)



Word Matching with a Trie (cont'd)



Word Matching with a Trie (cont'd)



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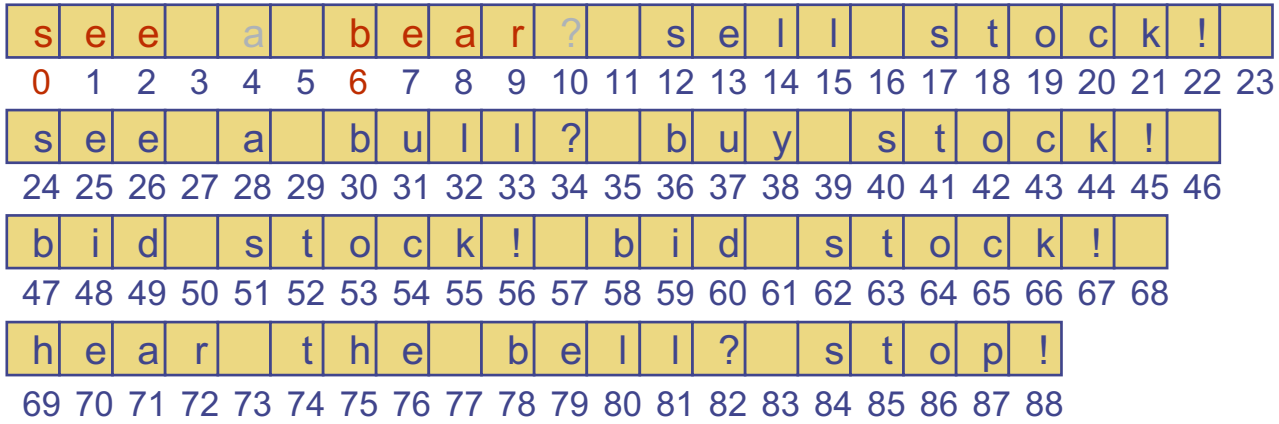
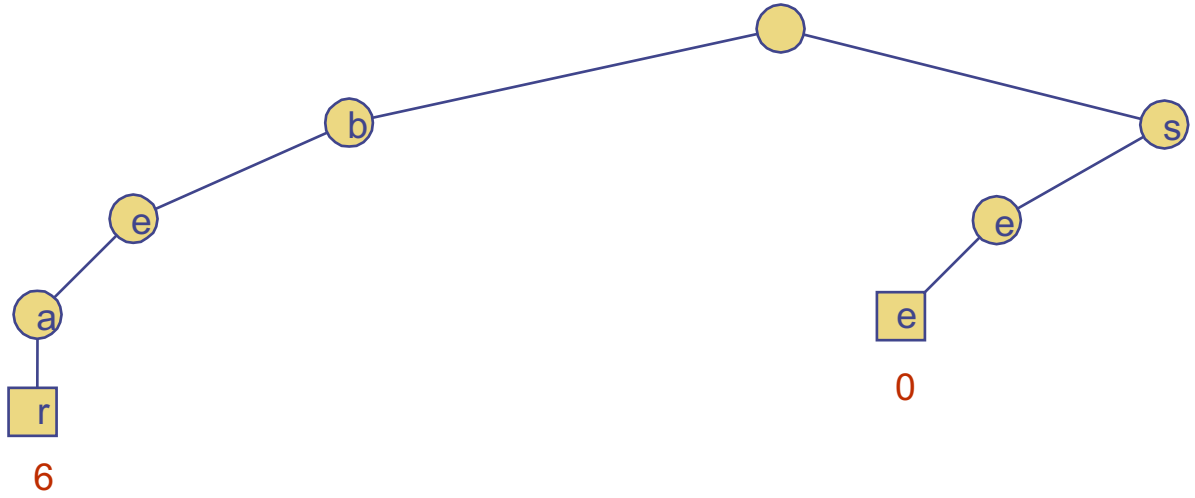
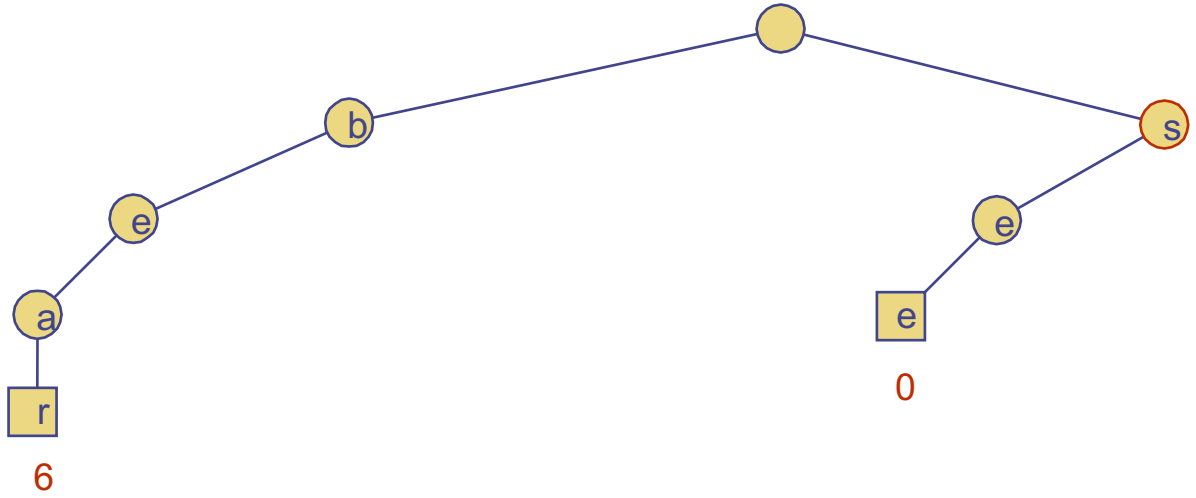




Word Matching with a Trie (cont'd)



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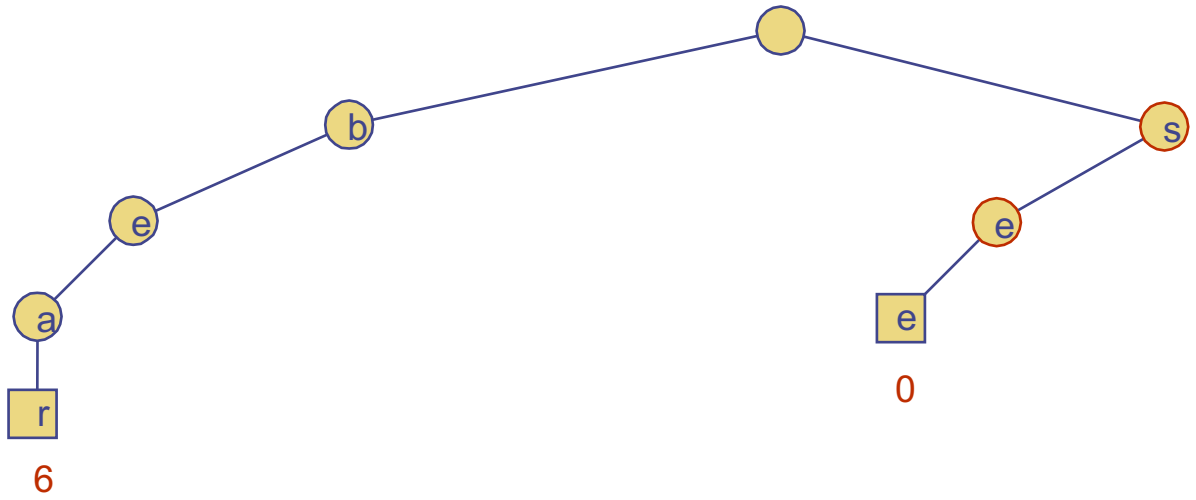

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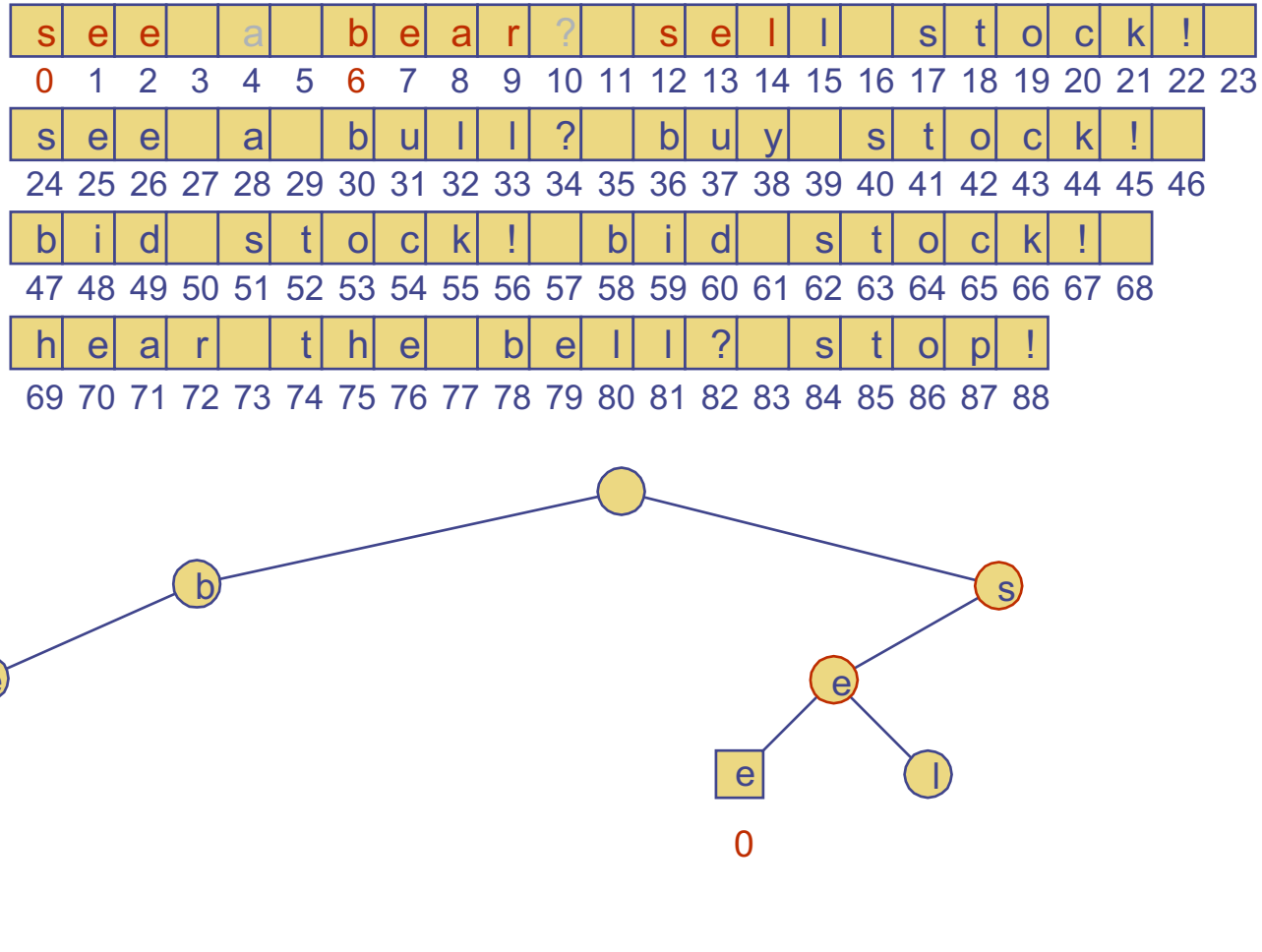

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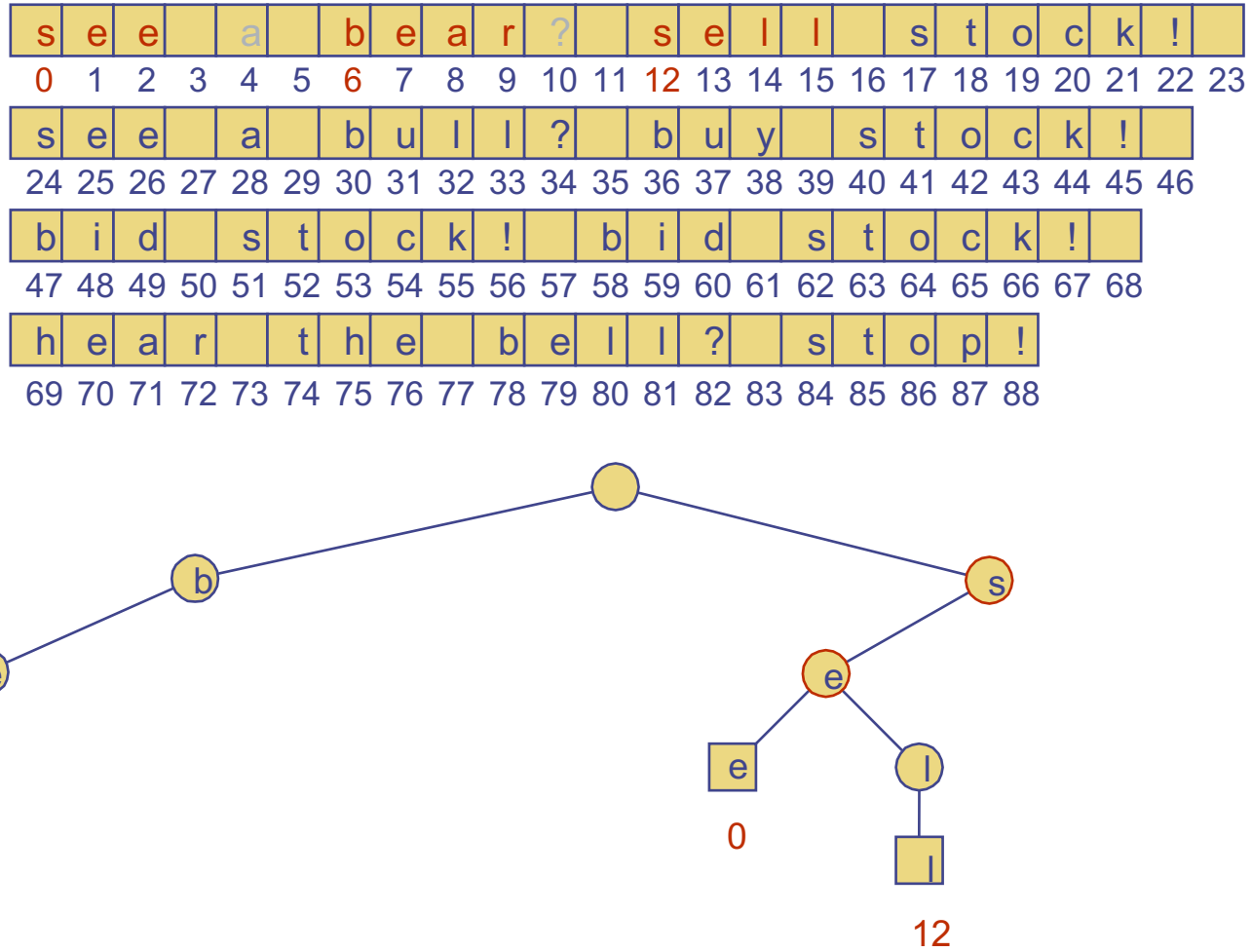

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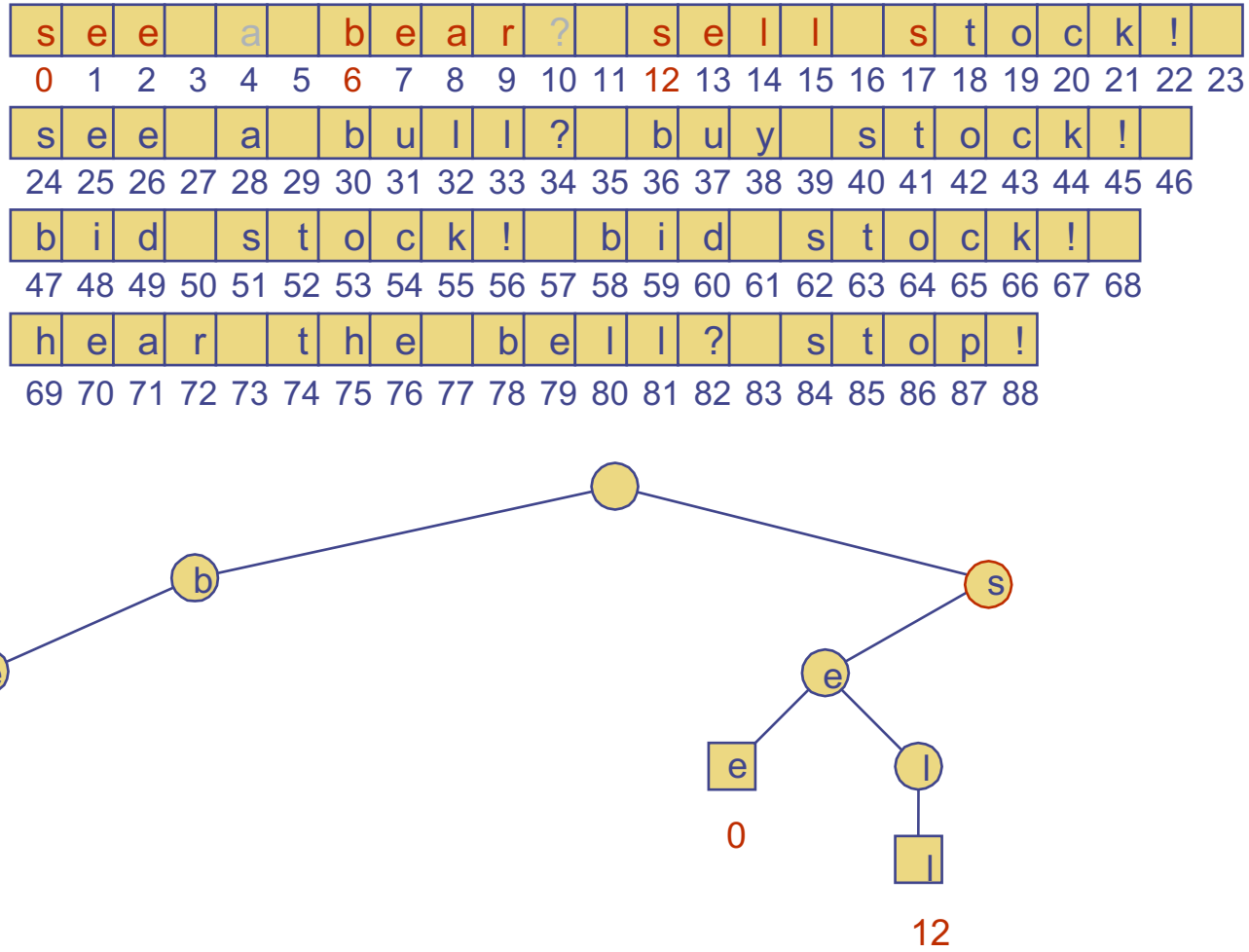

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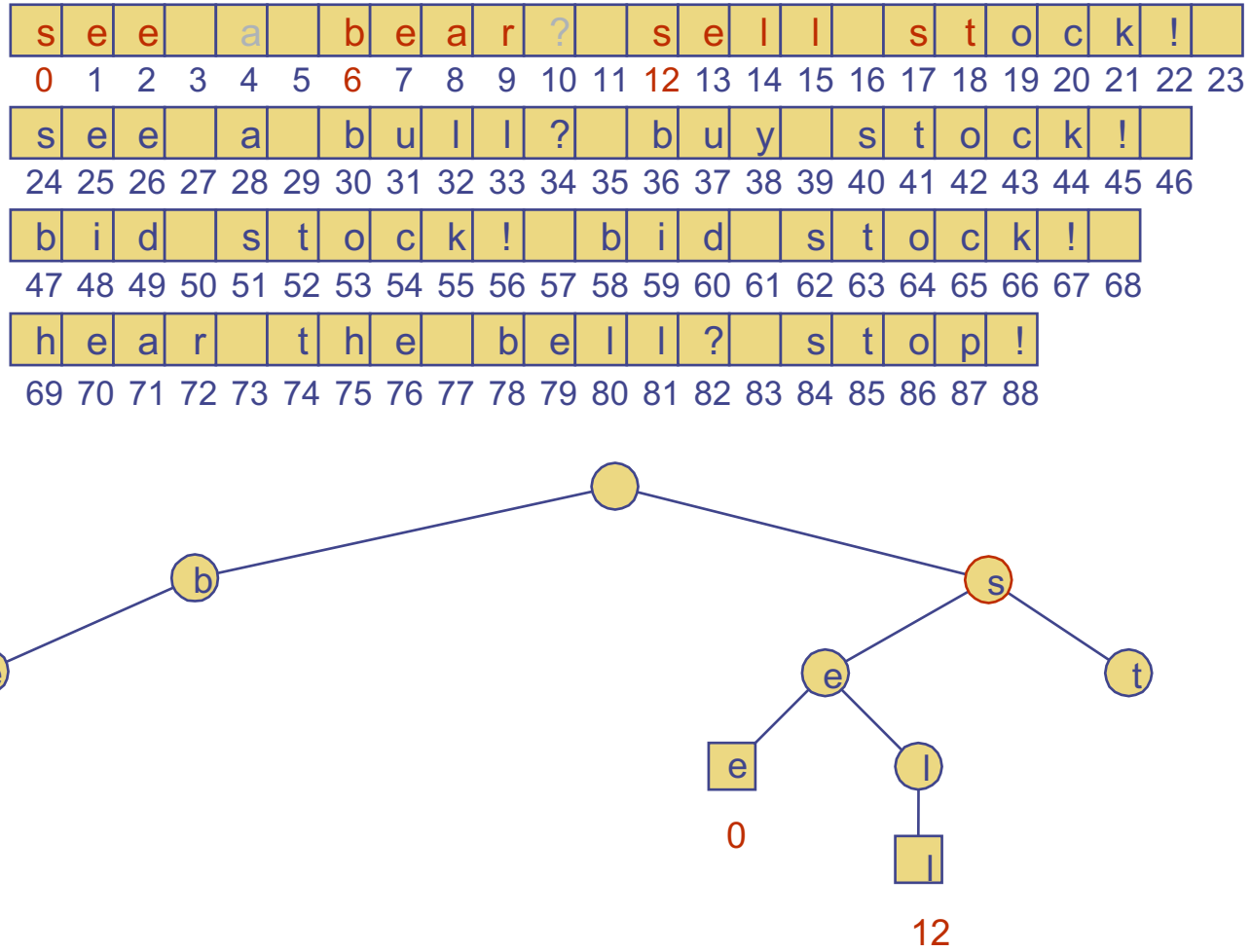

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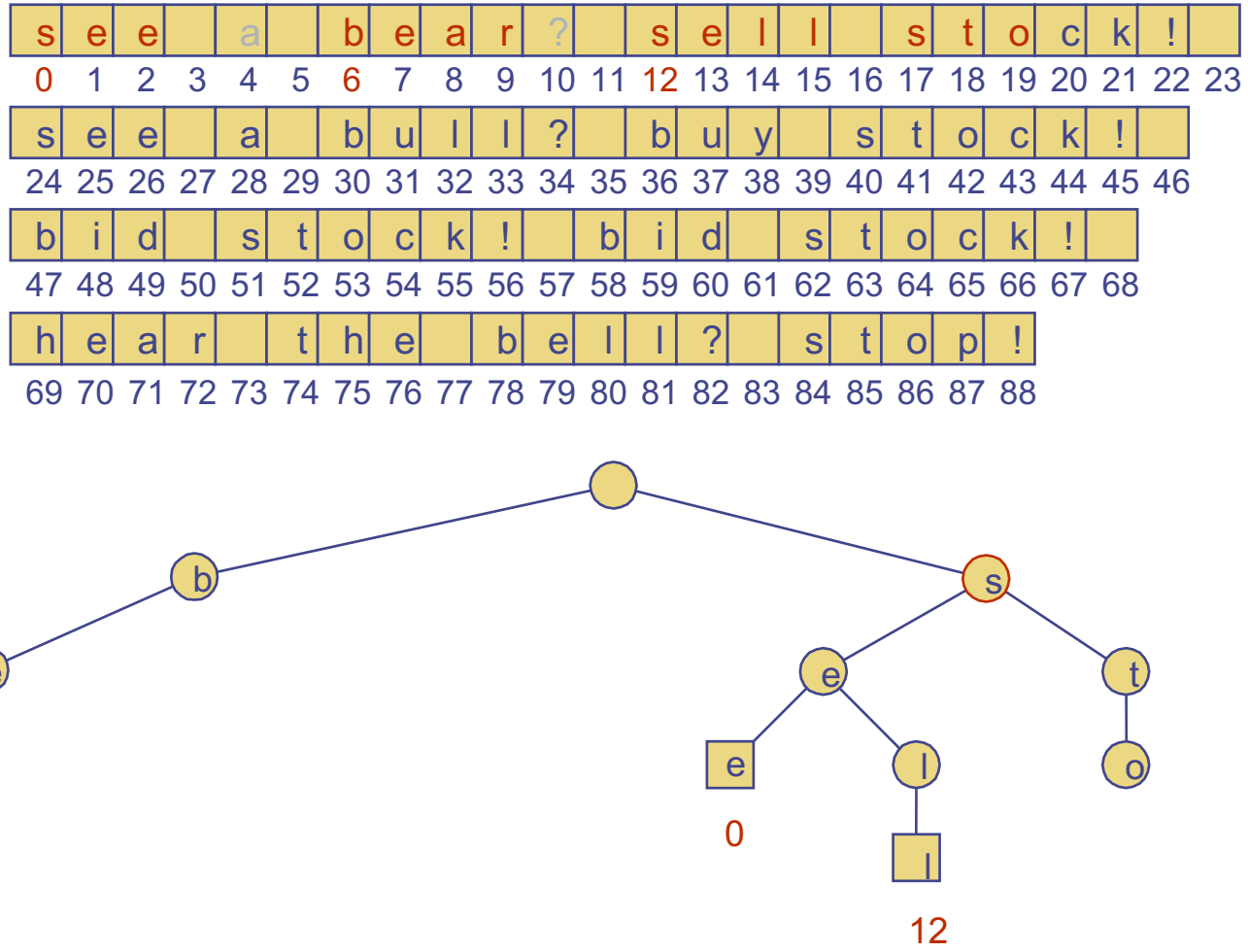

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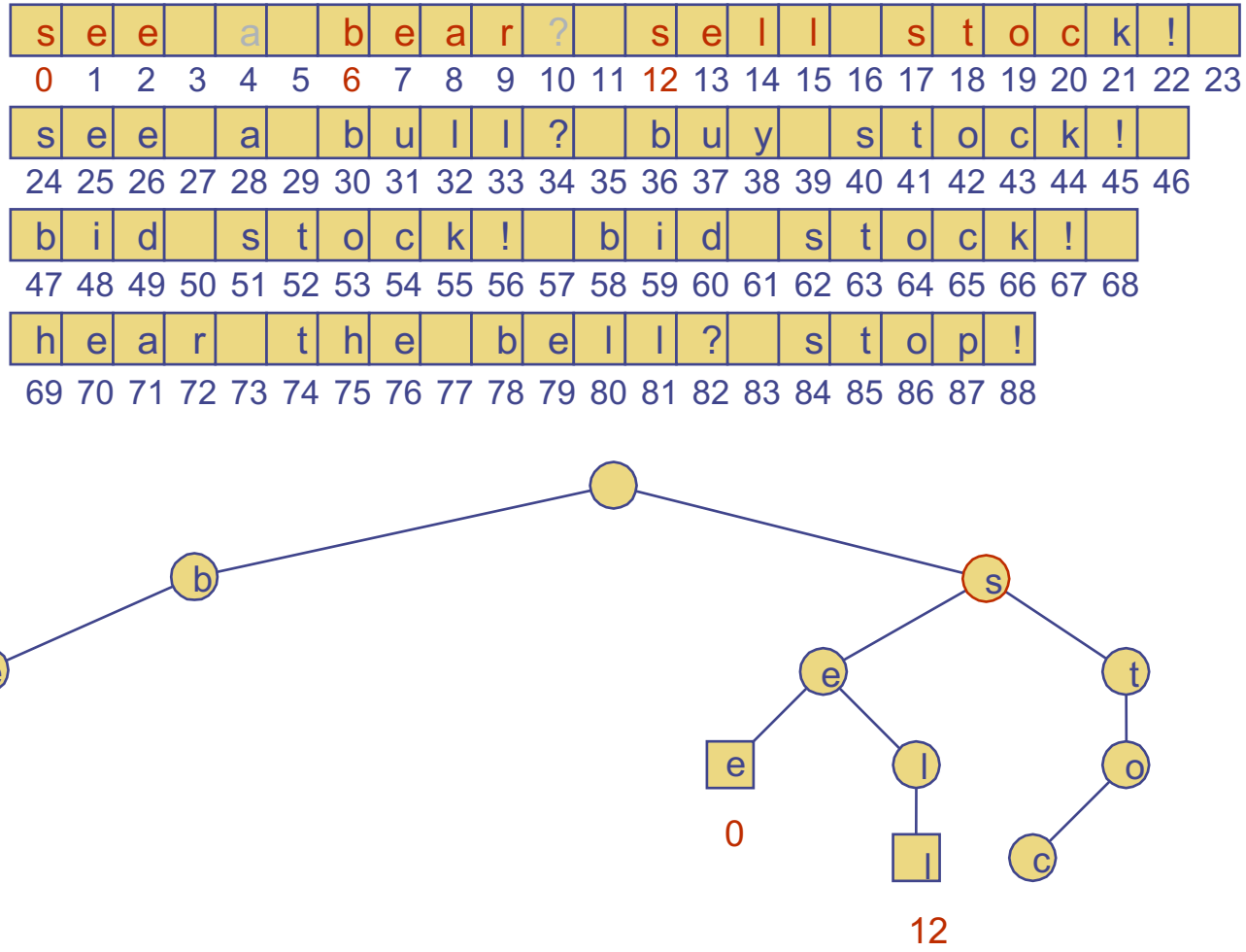

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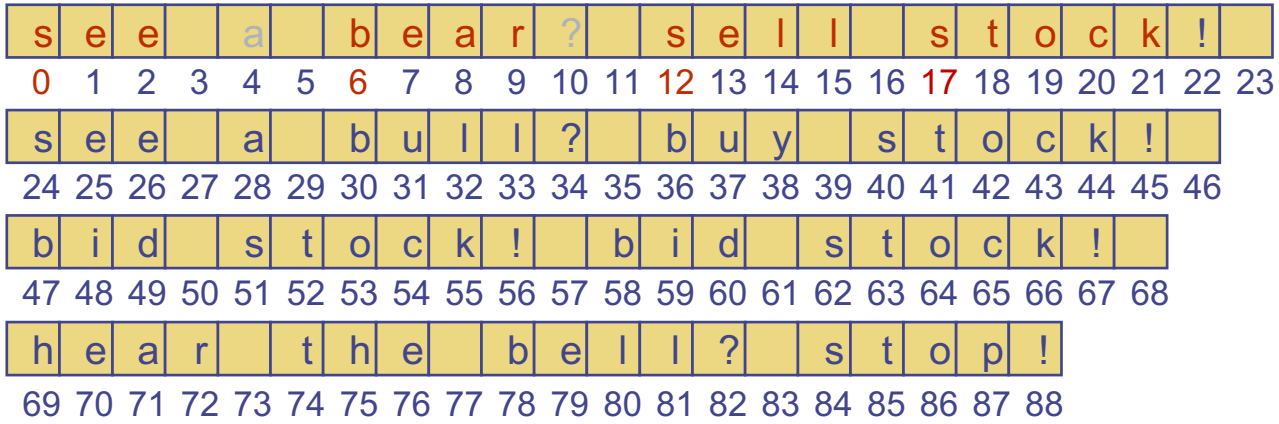
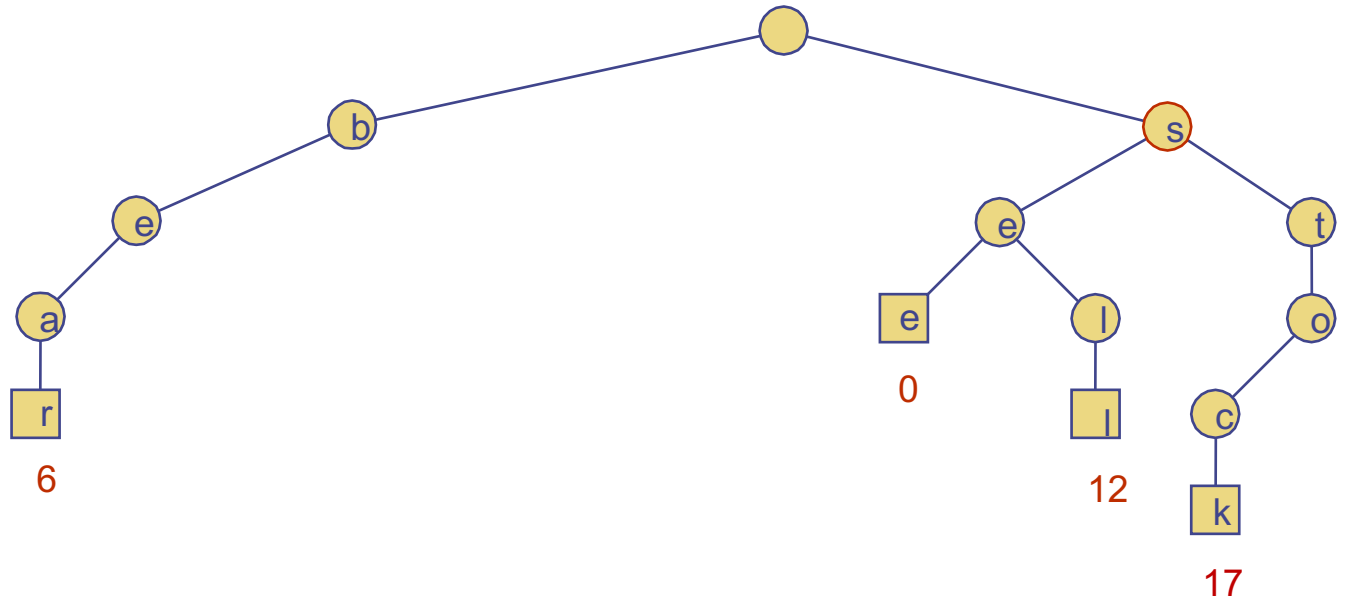
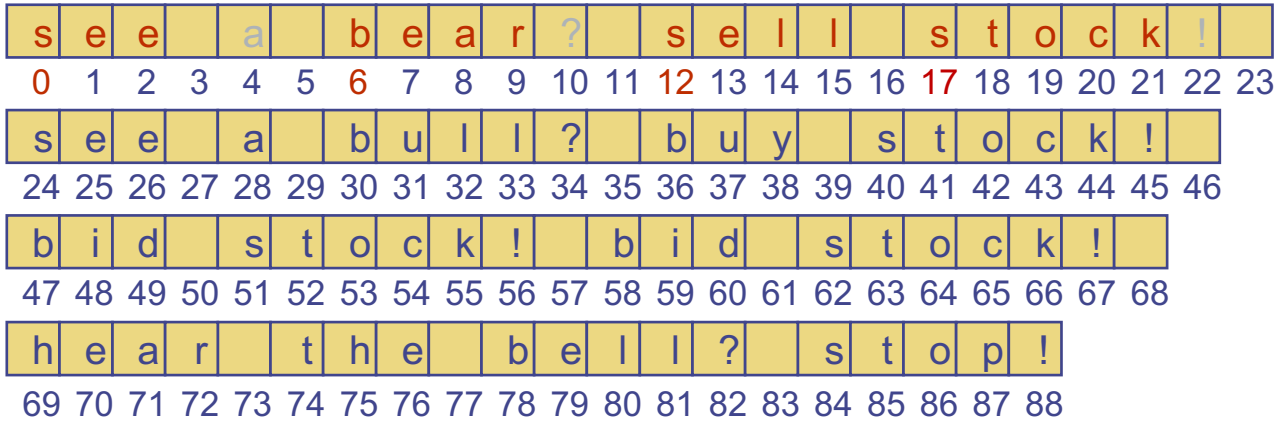
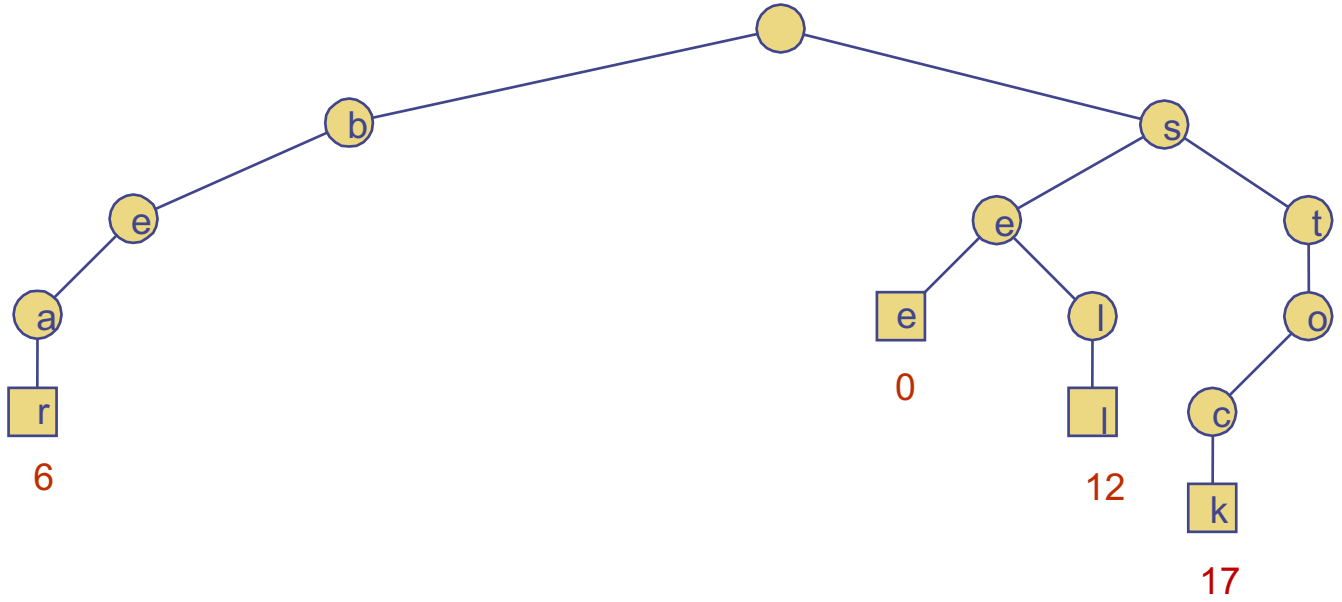




Word Matching with a Trie (cont'd)





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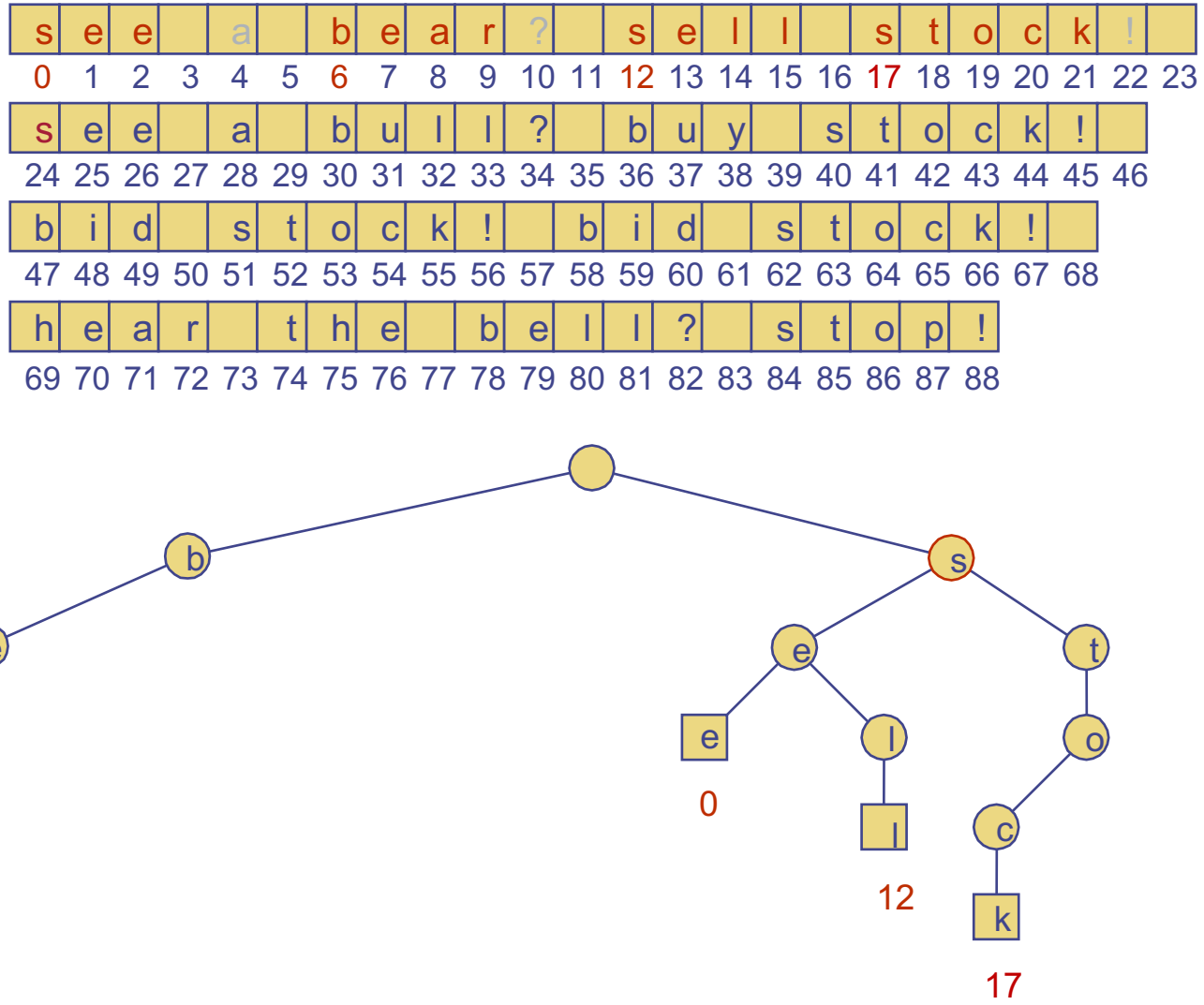

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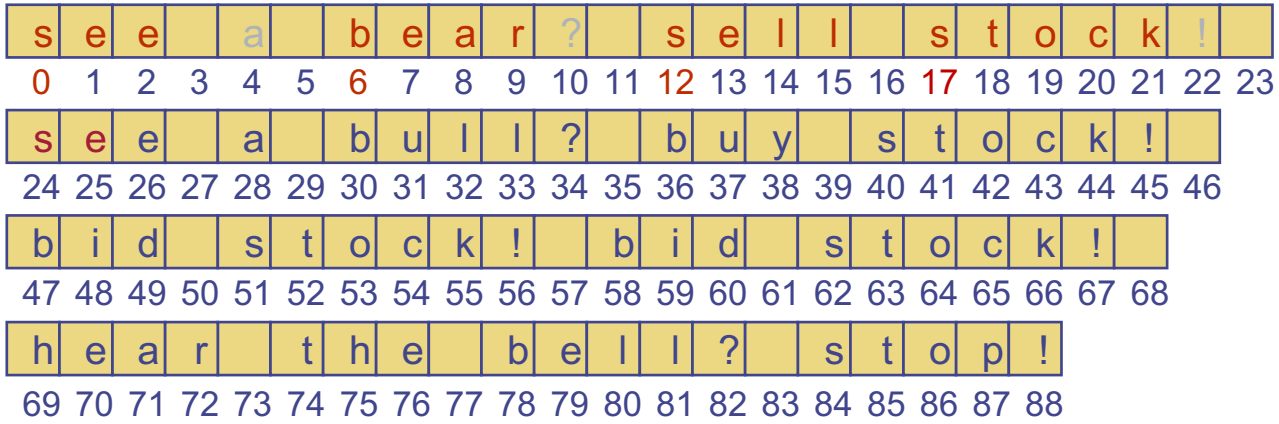

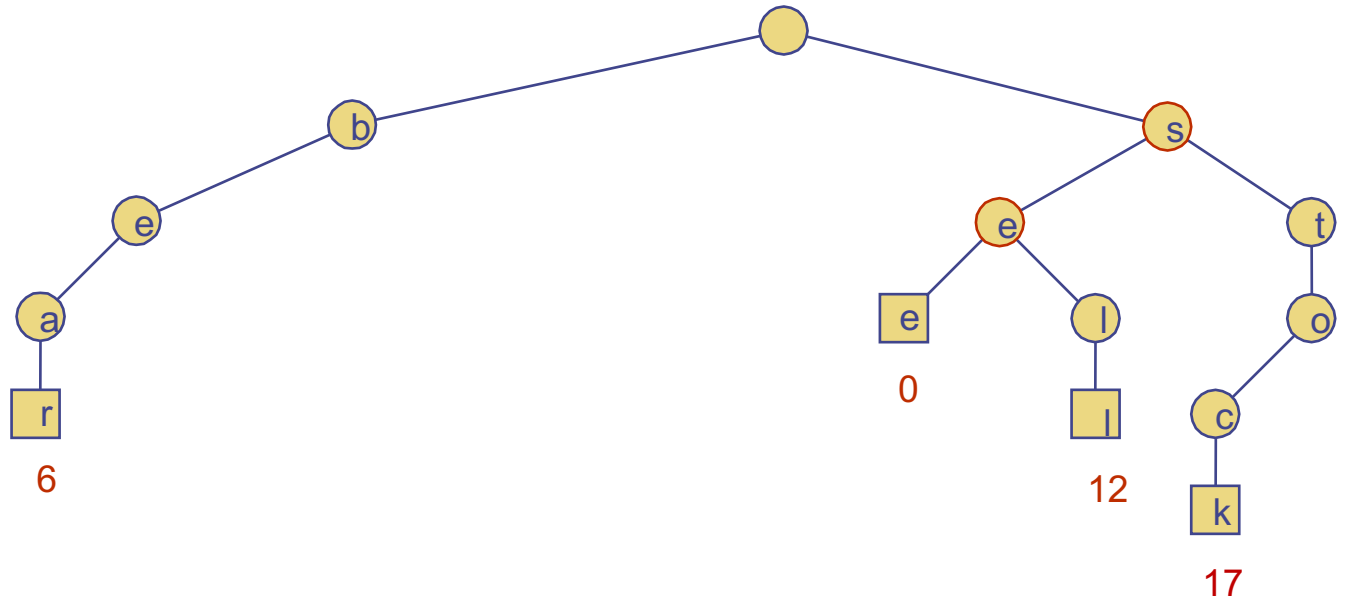

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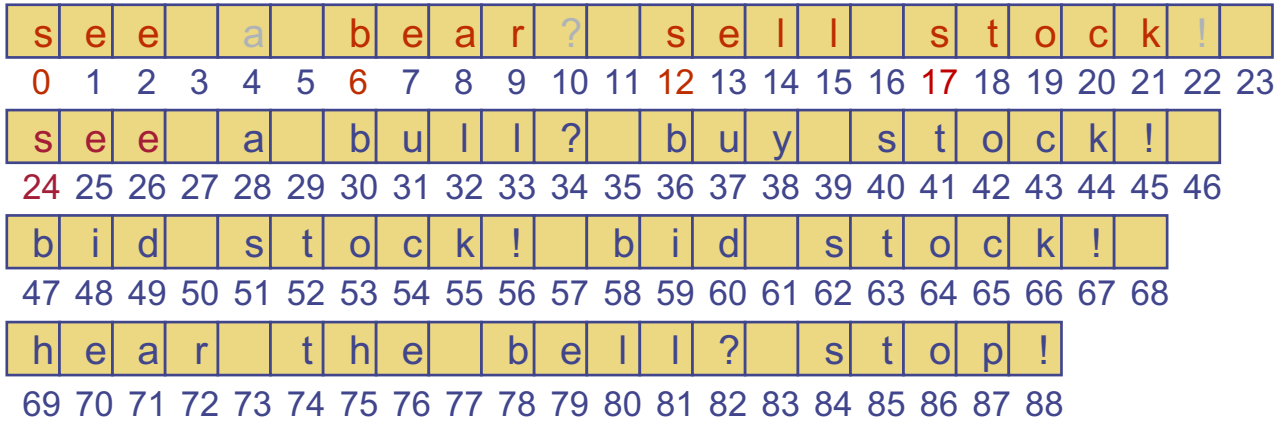

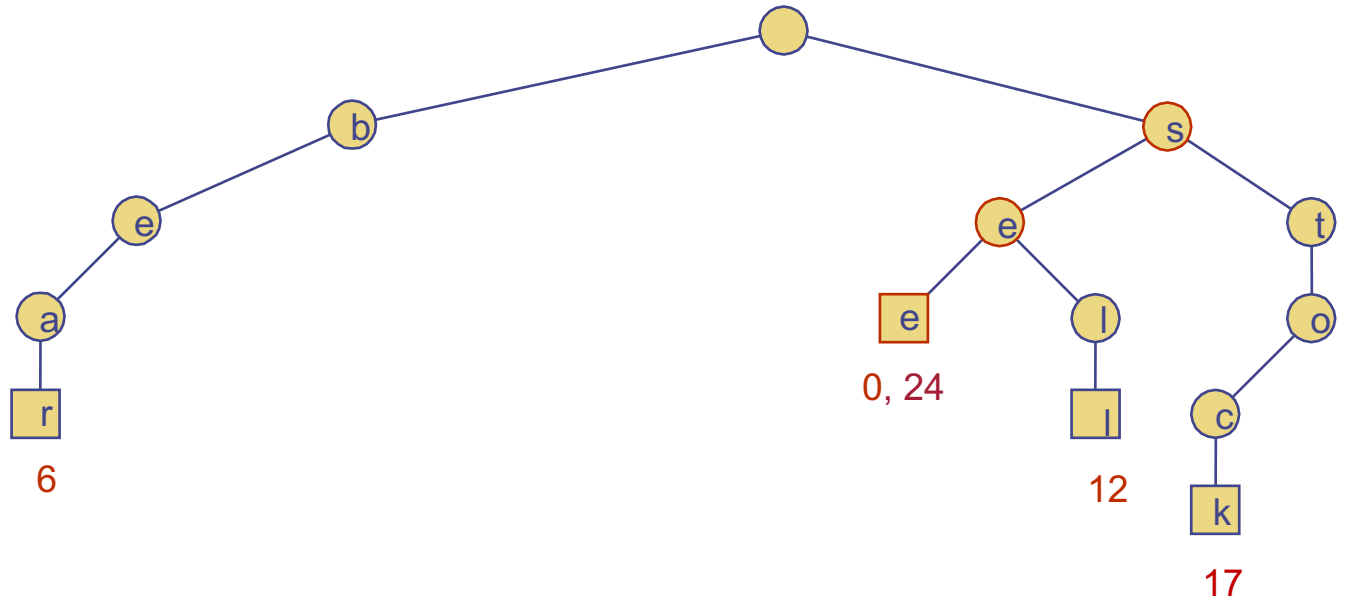

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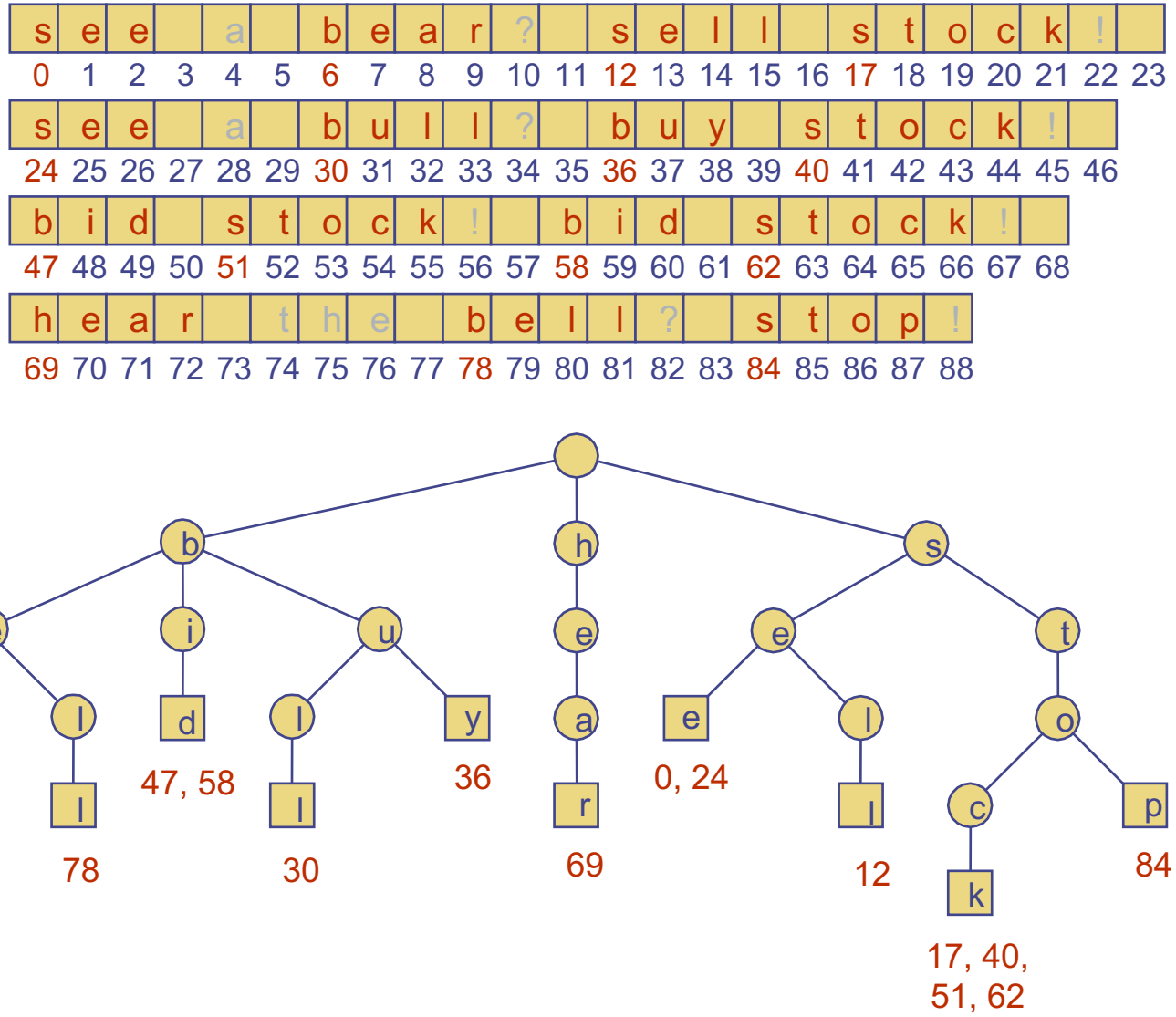
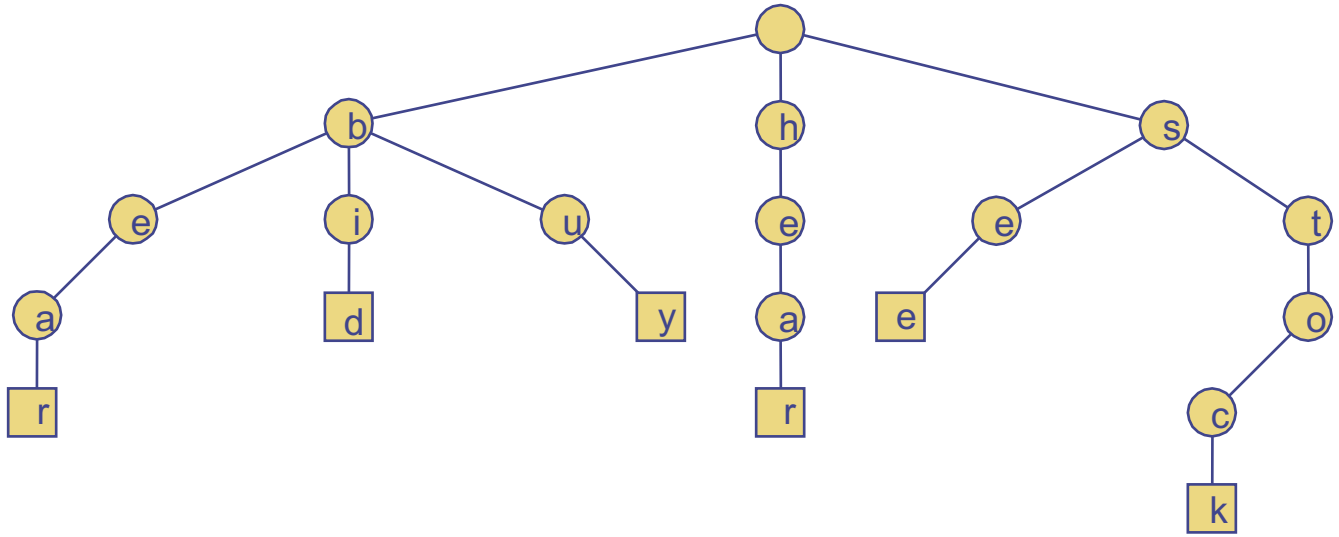


Standard Trie construction – Running Time

- construct a standard trie T for a set S of strings of total length n over the alphabet Σ
- Incremental algorithm
  - Insert strings one at a time
    - no string of S is a substring of another string
    - append \$ at the end of each string, where \$ $\notin |\Sigma|$, if there are prefixes
  - To insert the string X of size m into T: trace the path associated with X in T, creating new nodes if stuck
  - Running time for each insertion is similar to search: $O(m \cdot |\Sigma|)$ worst-case performance, $O(m)$ when using a secondary hash table at each node
  - Total expected running time: $O(n)$ - n is the total length of the strings in S

Better Tries?

- Standard tries have a potential space inefficiency
  - There can be many nodes with only one child, which is wasteful
- Solution
  - compressed tries (aka Patricia tries)

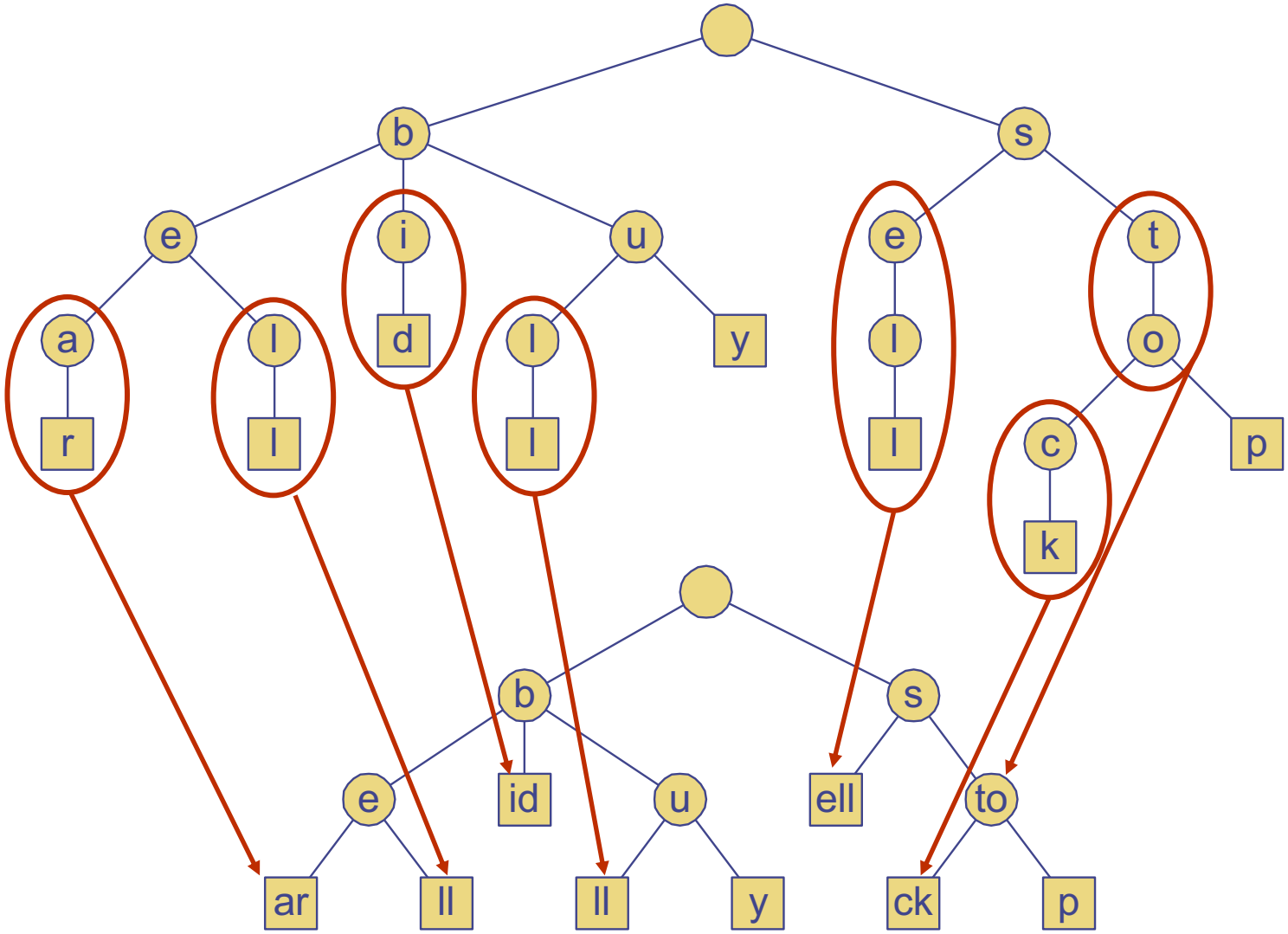


Compressed Tries

Compressed Tries

- Similar to a standard trie, but each internal node must have at least two children
- Rule enforced by compressing chains of single-child nodes into individual edges
- consider T, a standard trie; an internal node v of T is redundant if v has only one child and it is not the root
- A chain of $k \geq 2$ edges, $(v_0, v_1)(v_1, v_2) \dots (v_{k-1}, v_k)$ is redundant if:
  - v_i is redundant for $i = 1, \dots, k-1$
  - v_0 and v_k are not redundant
- A standard trie can be transformed into a compressed trie by replacing each redundant chain of $k \geq 2$ edges, $(v_0, v_1)(v_1, v_2) \dots (v_{k-1}, v_k)$, into a single edge (v_0, v_k) and relabeling v_k with the concatenation of the labels of the nodes $v_1, \dots, v_k$

Standard Trie → Compressed Trie Example



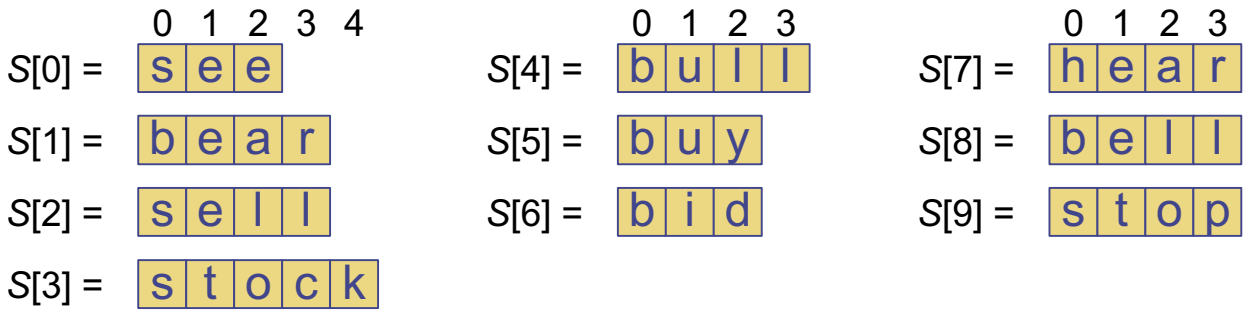
Properties of Compressed Tries

- in a compressed trie T storing a collection S of s strings from an alphabet of size $|\Sigma|$
  - Every internal node of T has at least two children and at most $|\Sigma|$ children
  - T has s leaves
  - The number of nodes in T is $O(s)$
    - Remember from the standard trie: the number of nodes of T was at most $n + 1$
    - Worst case: no two strings share a common, non-empty prefix – i.e. except for the root, all internal nodes have only one child
    - The worst-case scenario corresponds then to a compressed trie where all the chains from the root are redundant - and are therefore compressed – leading to $O(s)$ nodes

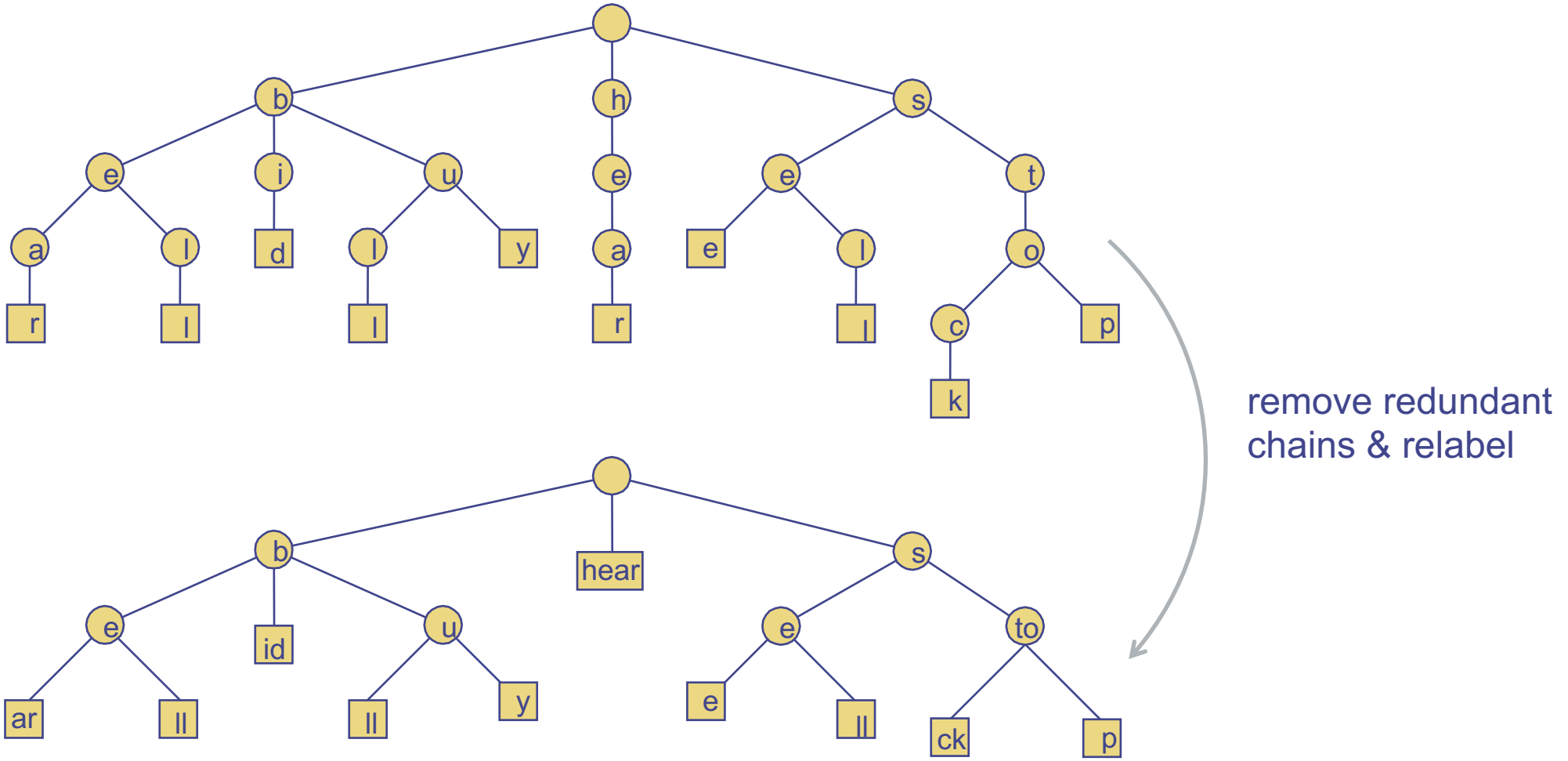
Compact Representation

- A compressed trie stores the node labels, which can be large (and have redundancy)
- Does the compression of the paths really provide an advantage?
- Yes, when using the compressed trie as an auxiliary index structure – not storing the actual characters, only indices into an already stored collection of strings

Compact Representation - Example

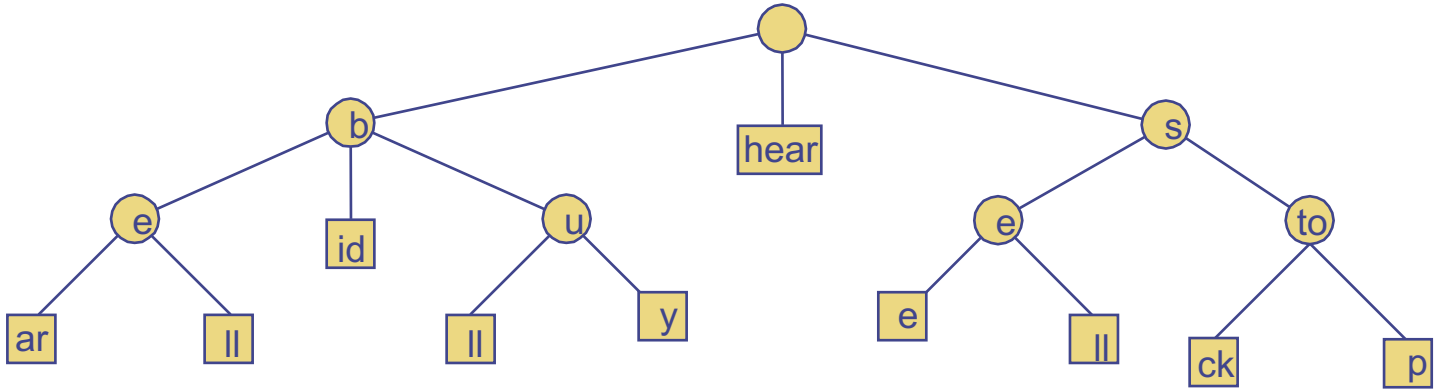


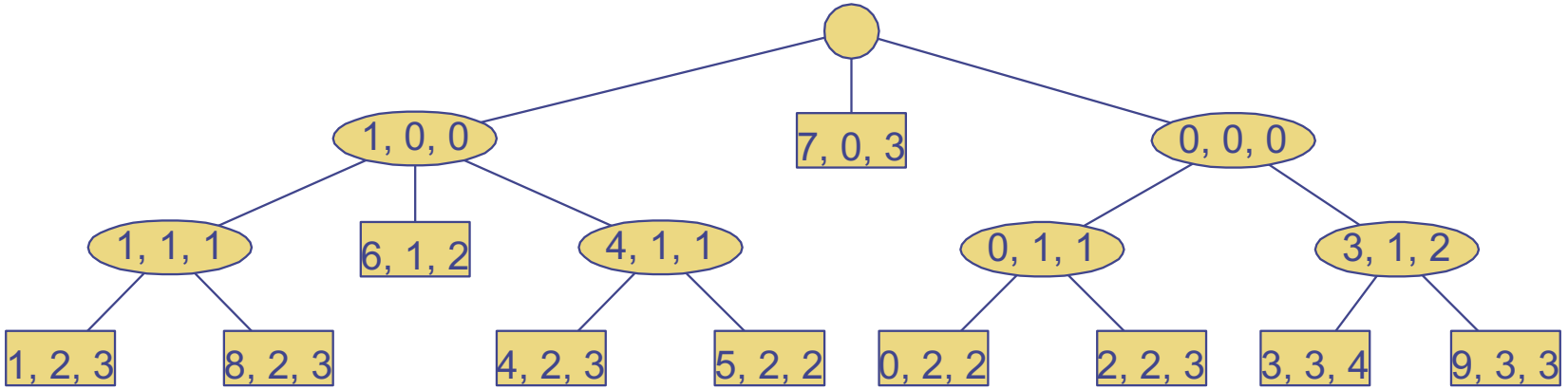
Compact Representation – Example (cont'd)



Compact Representation – Example (cont'd)

	0	1	2	3	4
$S[0]$ =	s	e	e		
$S[1]$ =	b	e	a	r	
$S[2]$ =	s	e	l	l	
$S[3]$ =	s	t	o	c	k
$S[4]$ =	b	u	l	l	
$S[5]$ =	b	u	y		
$S[6]$ =	b	i	d		
$S[7]$ =	h	e	a	r	
$S[8]$ =	b	e	l	l	
$S[9]$ =	s	t	o	p	





Compact representation gains

- Using the indexing method the total space required for storing the trie is reduced from $O(n)$ – where n is the total length of the strings in S - to $O(s)$ – where s is the number of strings in S
- The strings themselves must be stored in an additional structure, but we reduce the space required for the trie - which we need for searching
- Searching in a compressed trie is not necessarily faster than in a standard trie
  - Every character of the searched string must be compared with the character labels while traversing the trie
  - Labels can be multi-character

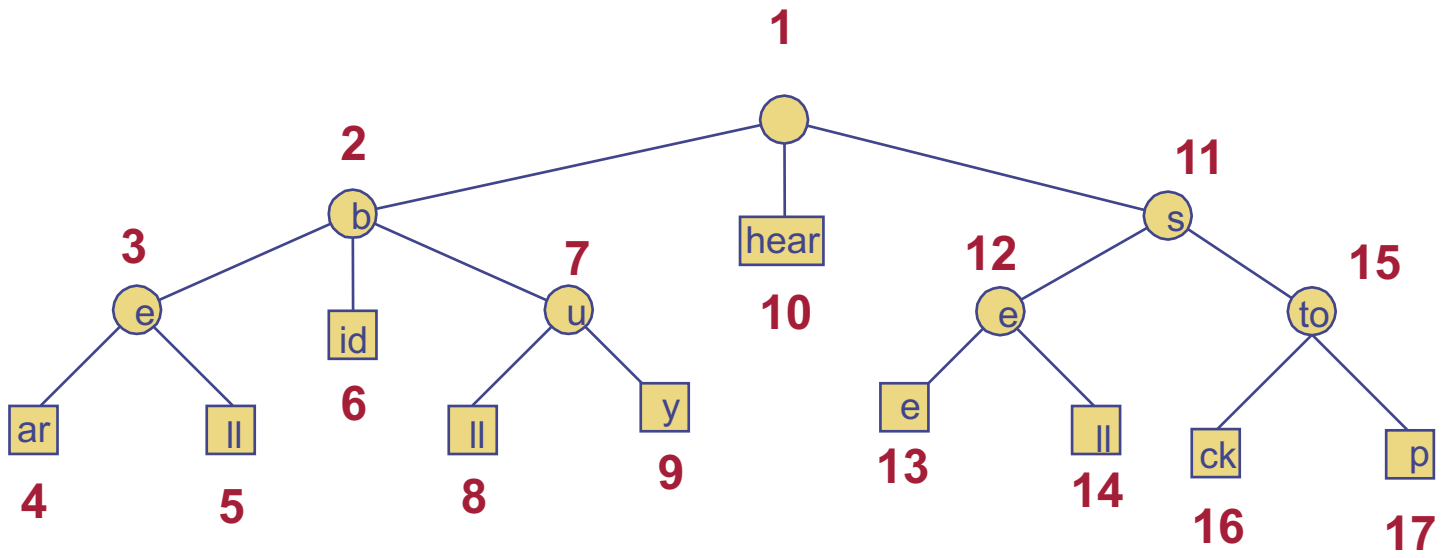
Tree Traversal

- to compress the nodes, one would need to traverse the trie (which is a tree)
- a traversal visits the nodes of a tree in a systematic manner
- types of traversals for n-ary trees
  - preorder traveral
  - postorder traversal

Preorder Traversal

- in preorder traversal, a node is visited before its descendants

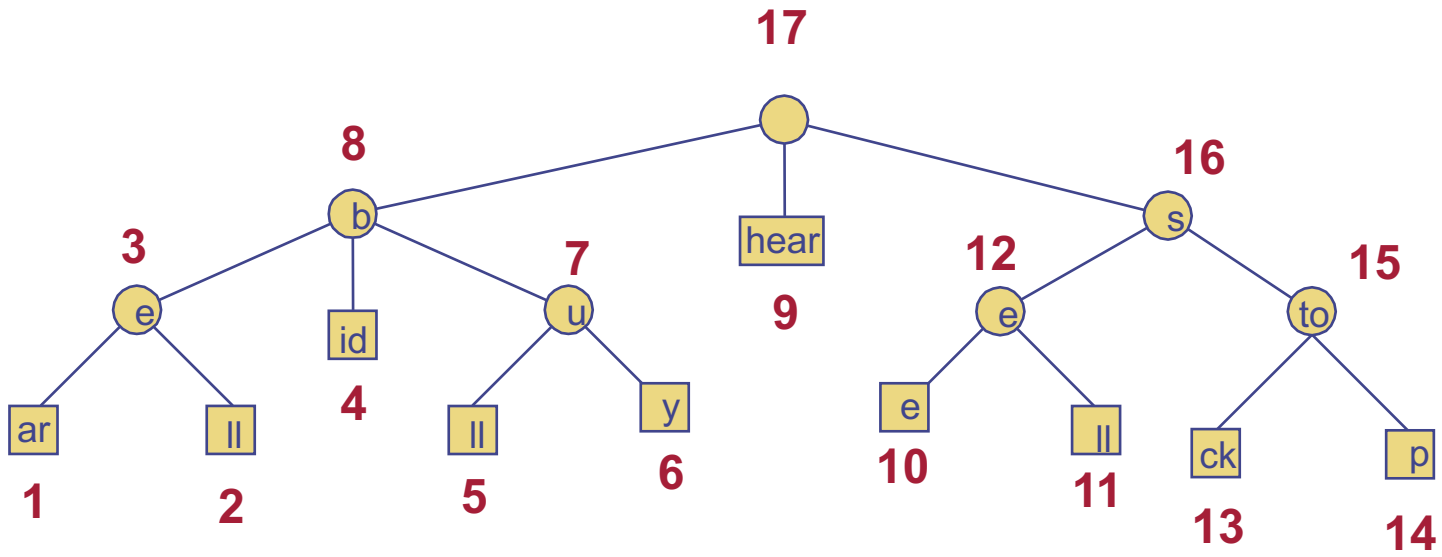
Algorithm *preOrder*(*v*)
    visit(*v*)
    for each child *w* of *v*
        preorder (*w*)



Postorder Traversal

- in postorder traversal, a node is visited after its descendants

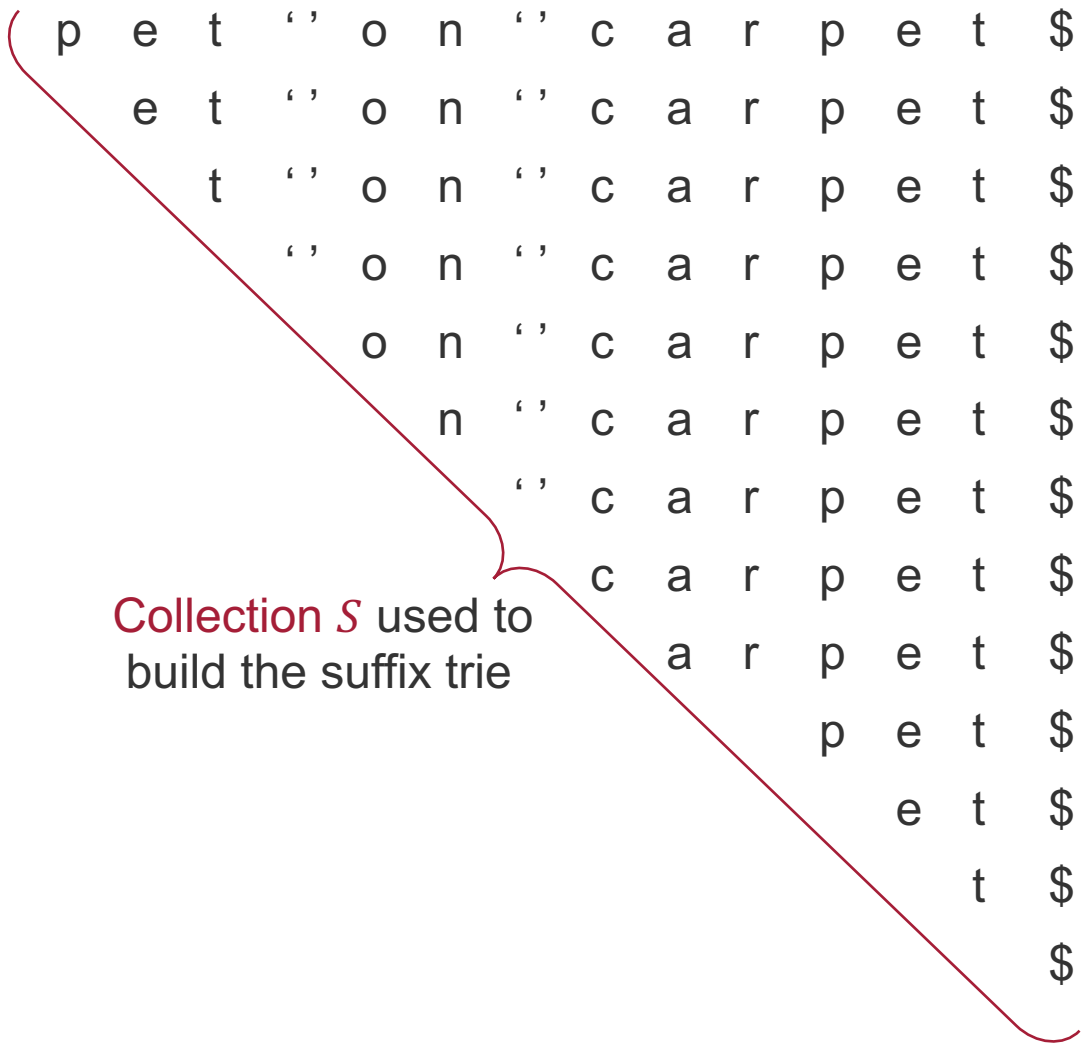
Algorithm *postOrder(v)*
  for each child *w* of *v*
    postOrder (*w*)
  visit(*v*)



Suffix Tries

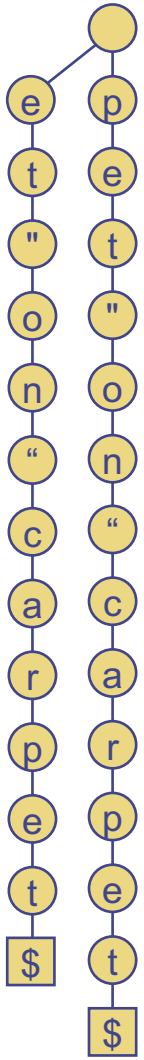
Suffix Tries

- The strings in the collection S are all suffixes of a string X
- Called a *suffix trie*, *suffix tree* or *position tree*
- Example X = *pet on carpet*
- S was supposed to be a set of s strings from alphabet Σ such that no string in S is a prefix for another string
- Add an artificial character \$, which is not part of Σ, at the end of each suffix
- For X of length n, we build a trie using the set of n strings $X[j:n]$, $j = 0 \ldots n-1$

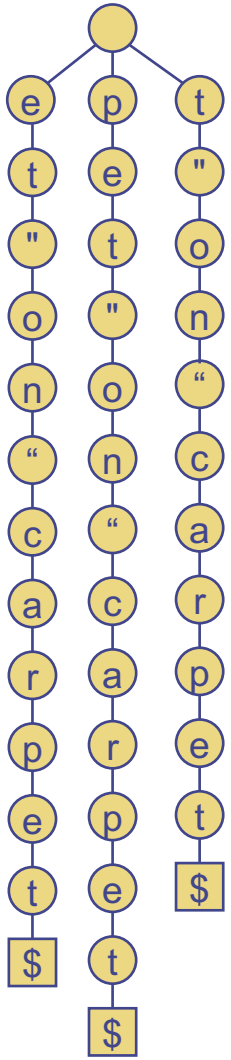


Suffix Trie Construction

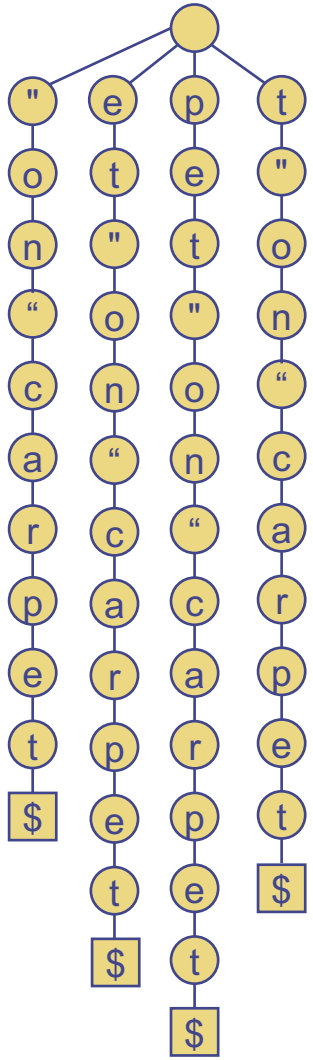
Suffix Trie Construction



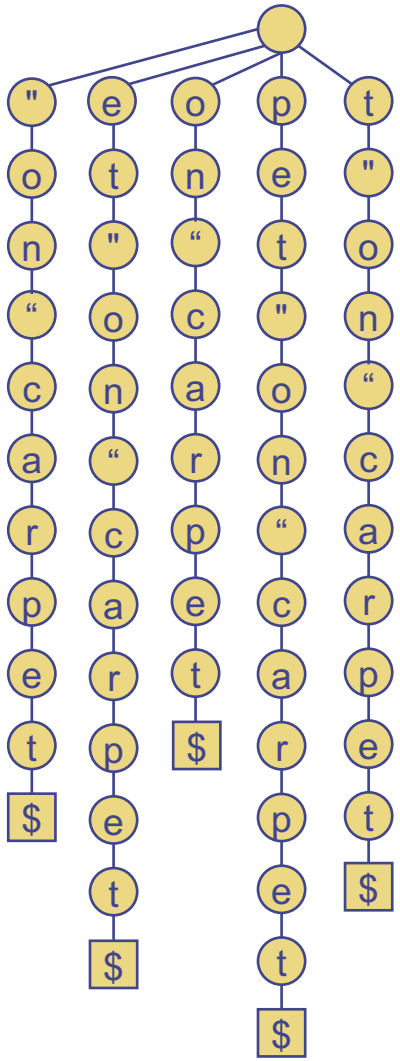
Suffix Trie Construction



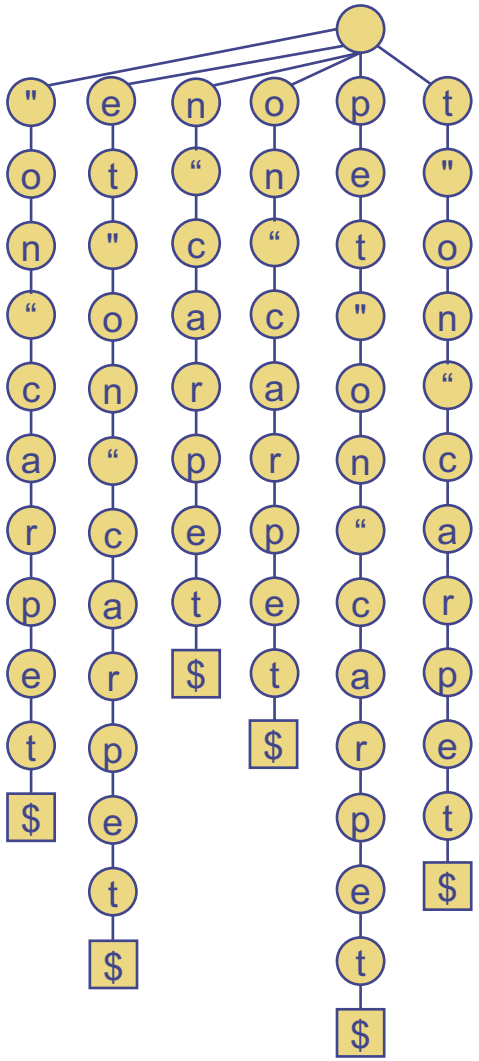
Suffix Trie Construction



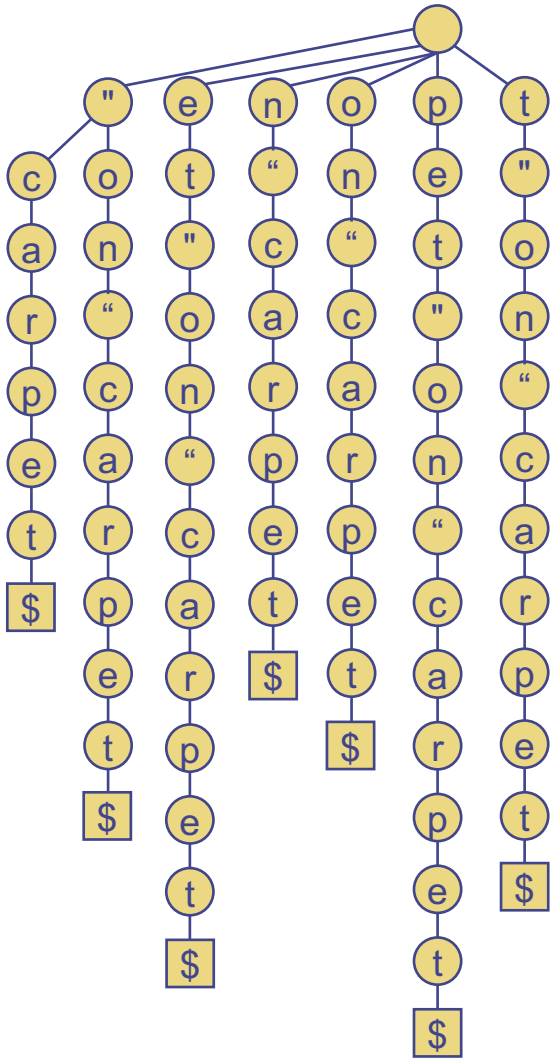
Suffix Trie Construction



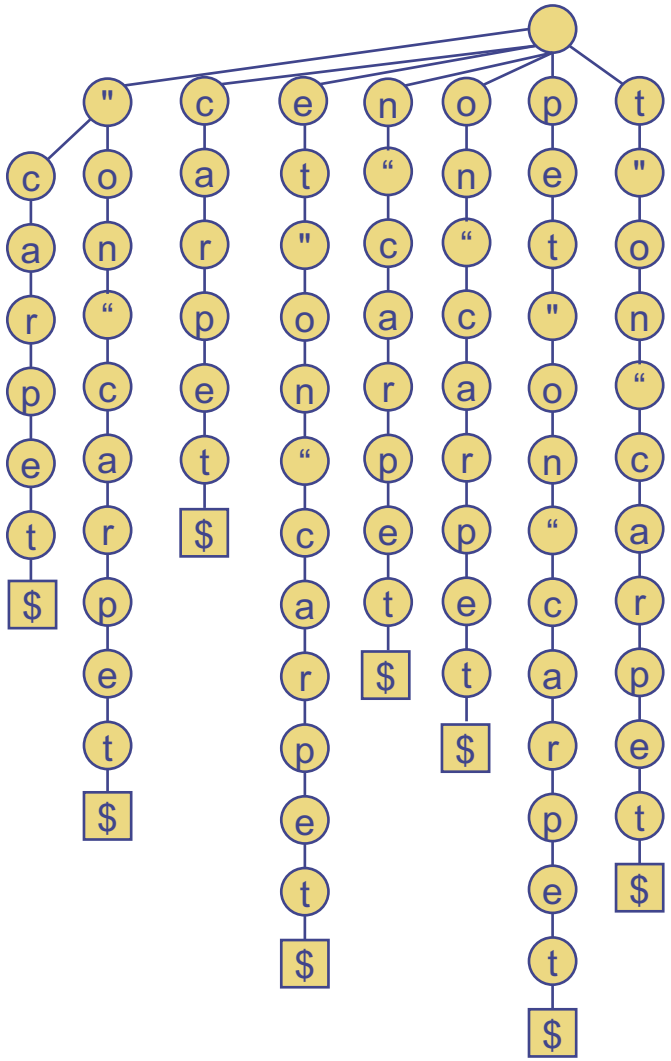
Suffix Trie Construction



Suffix Trie Construction



Suffix Trie Construction



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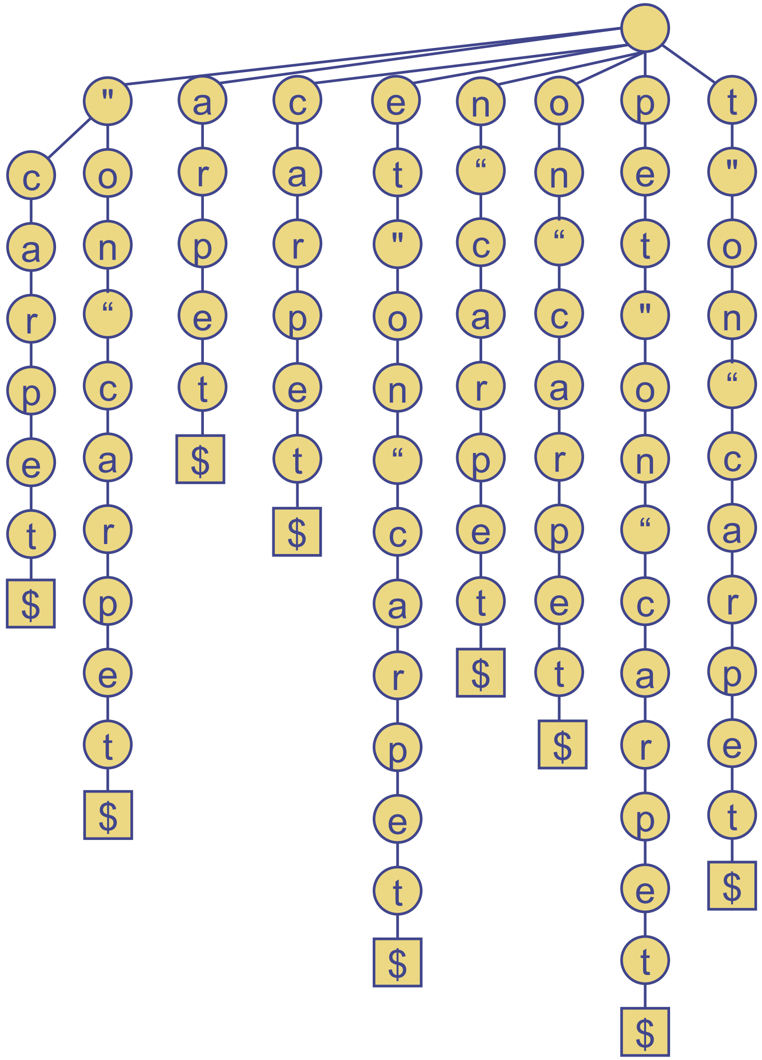
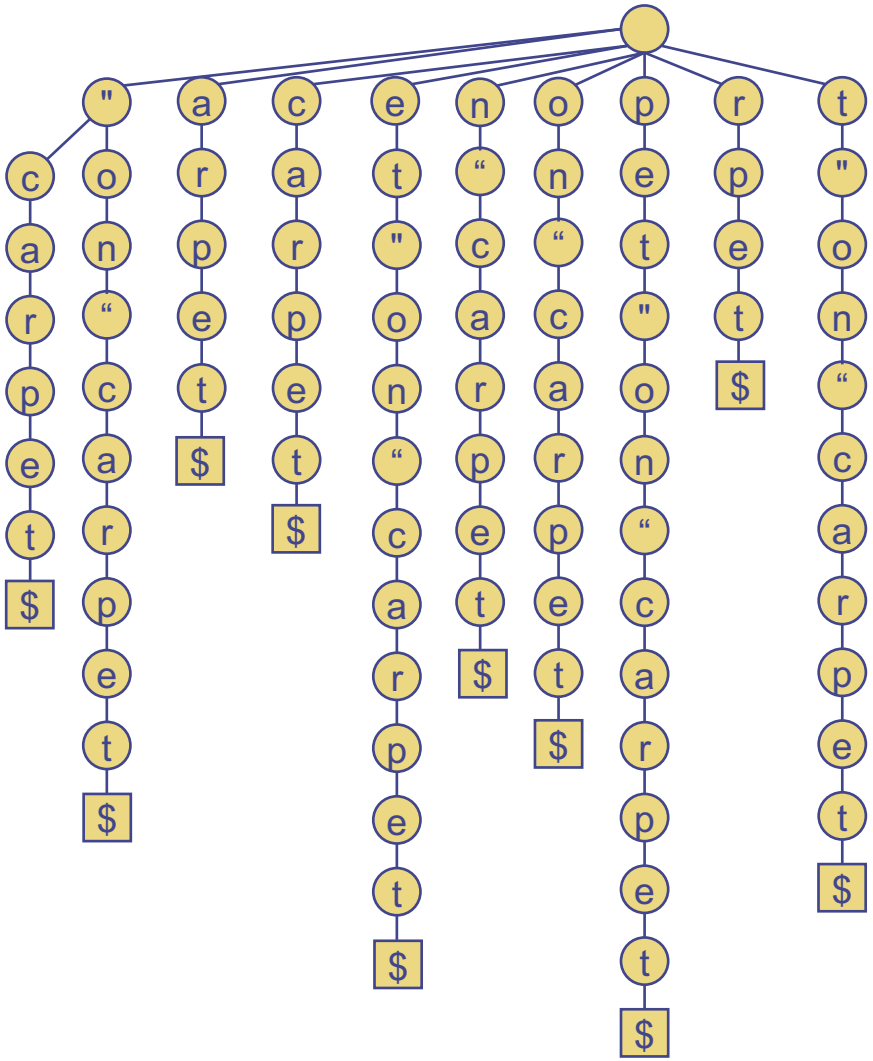
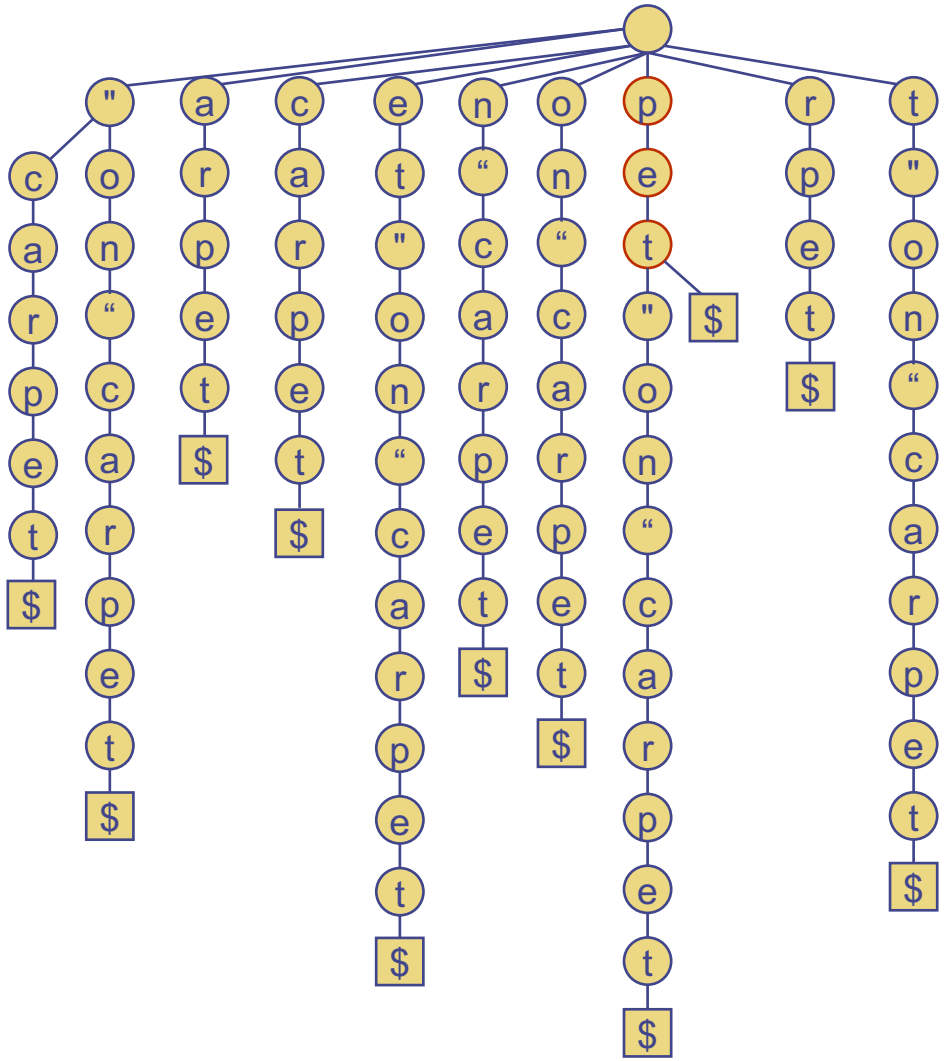


Suffix Trie Construction



Suffix Trie Construction



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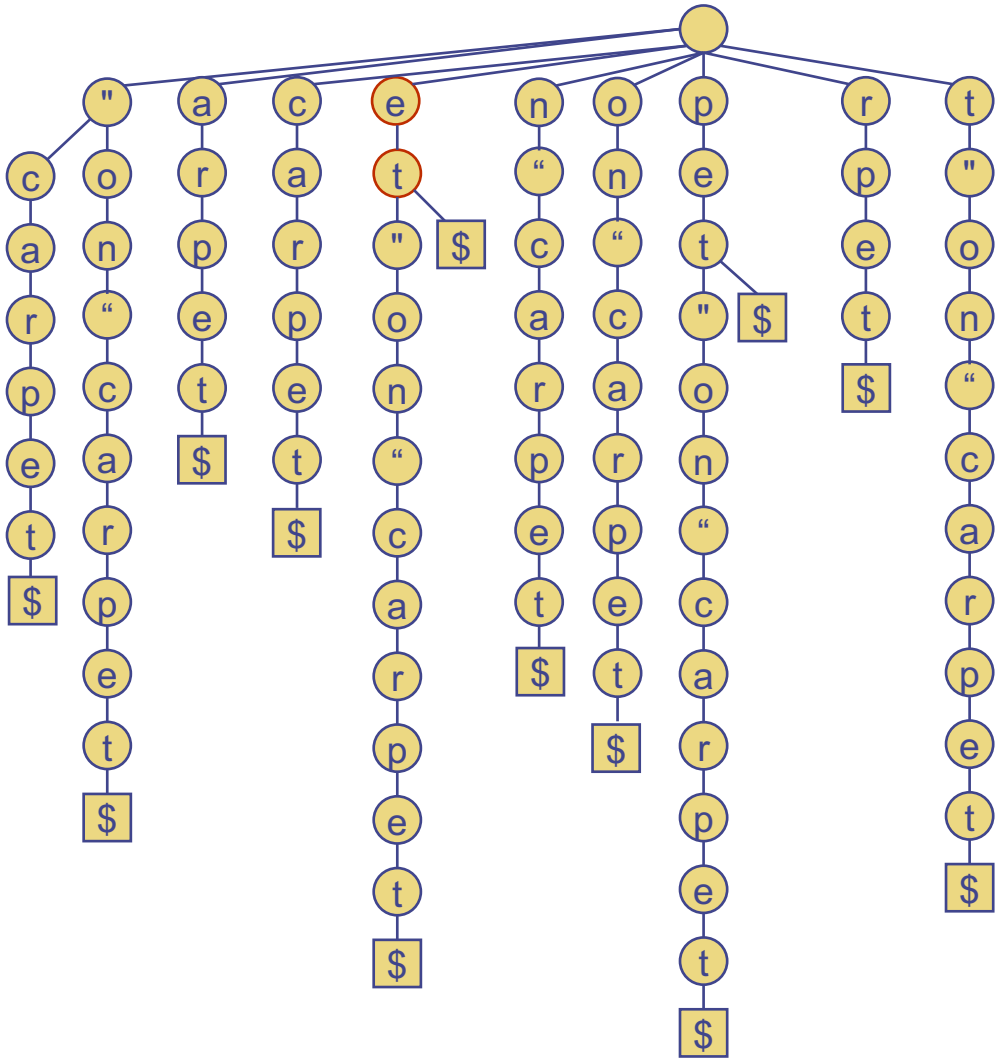
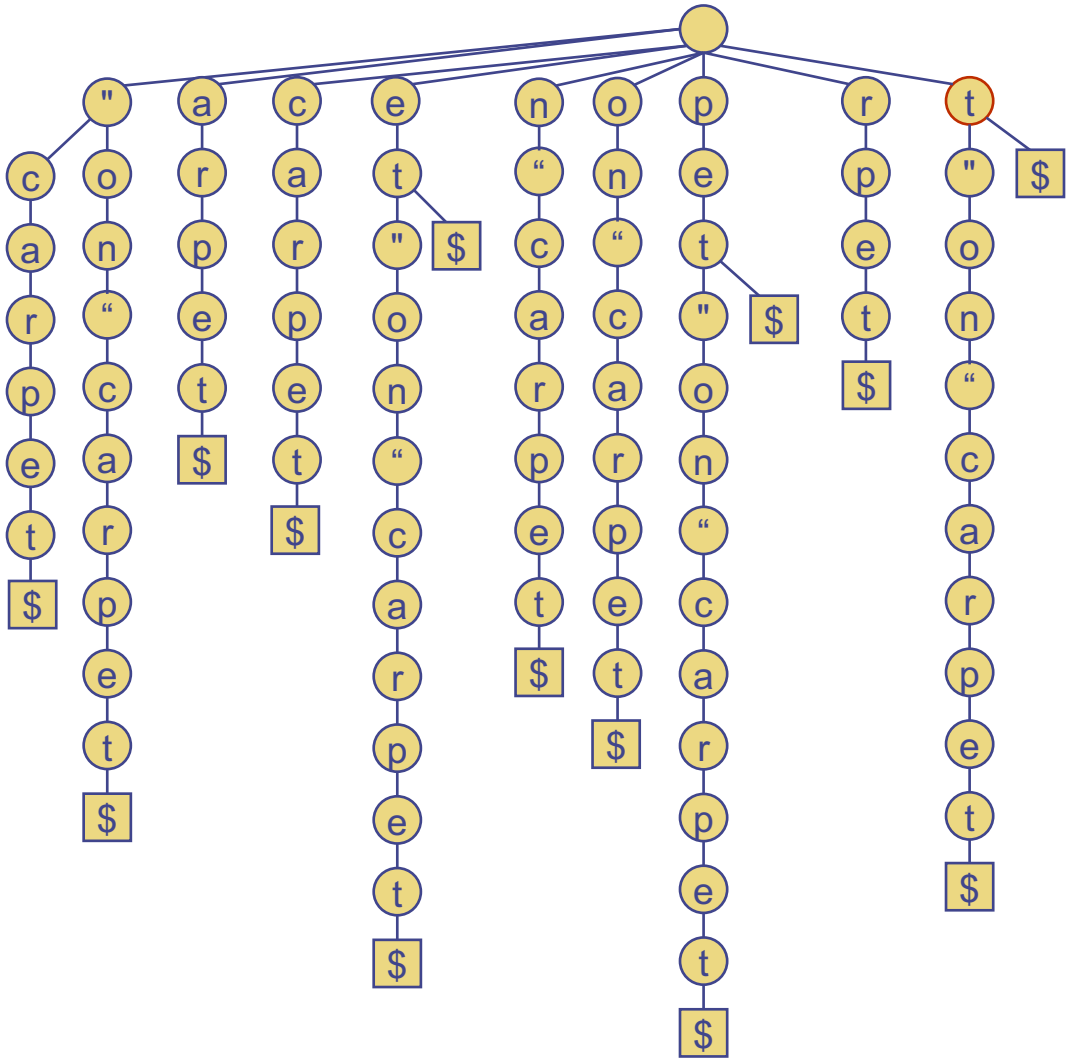
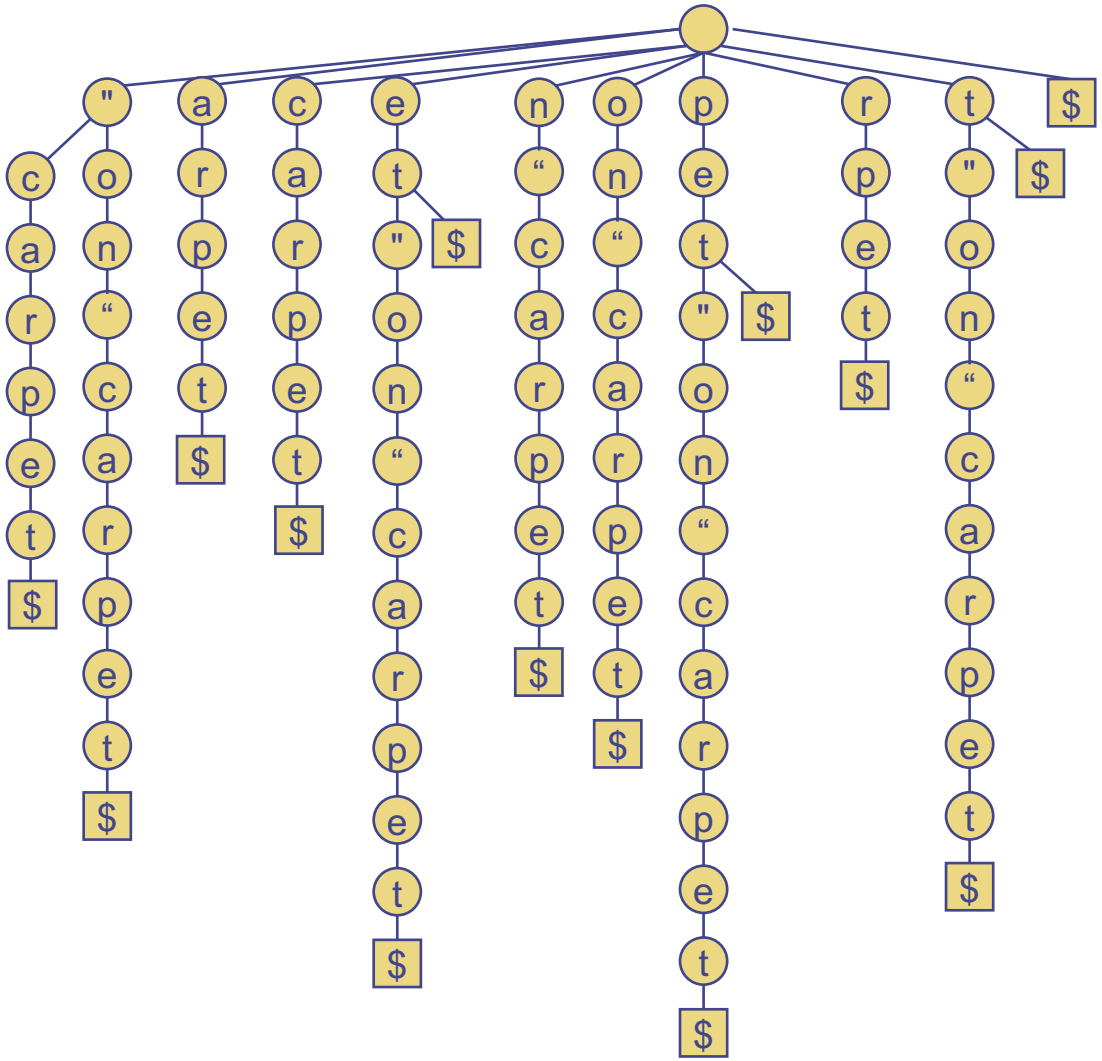


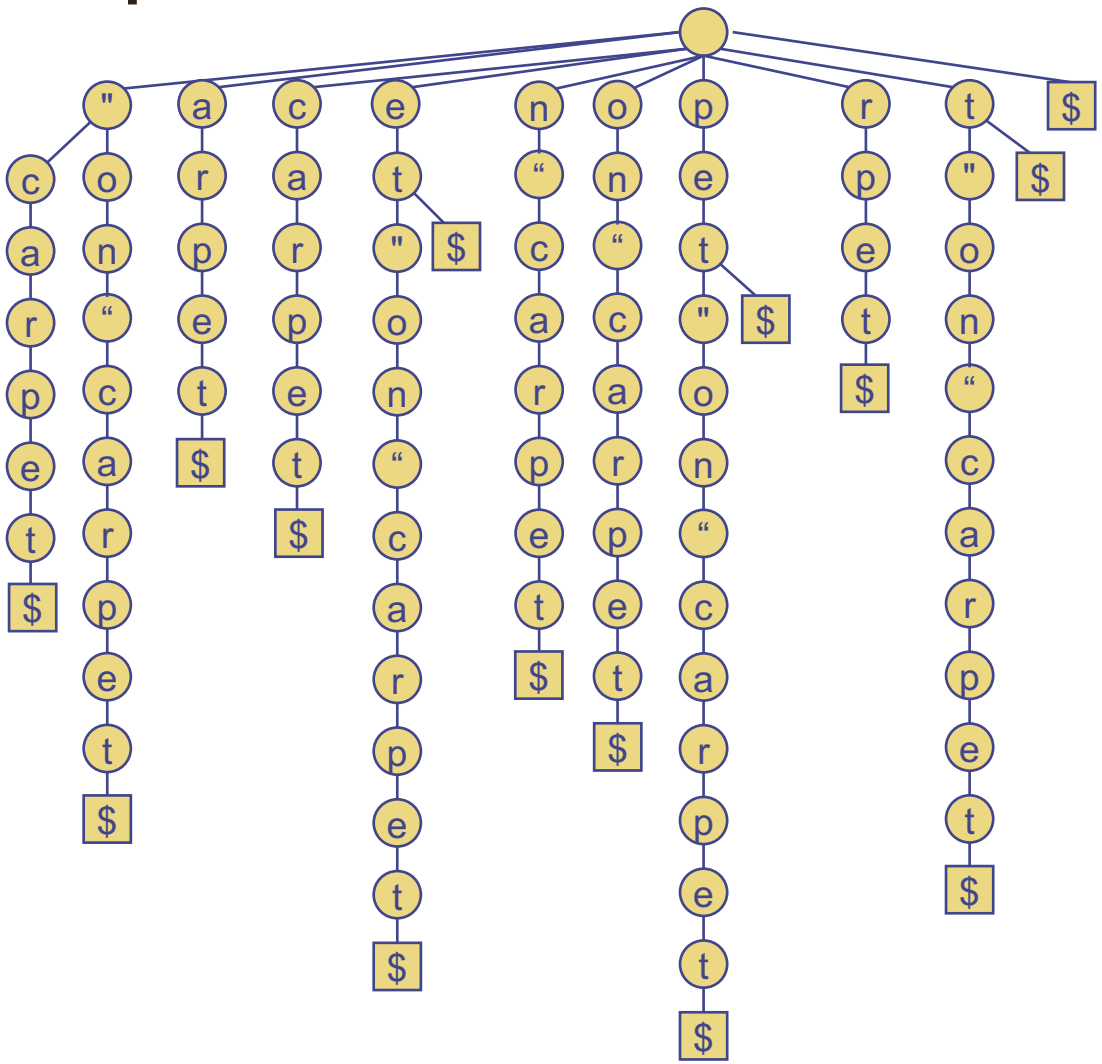
Suffix Trie Construction



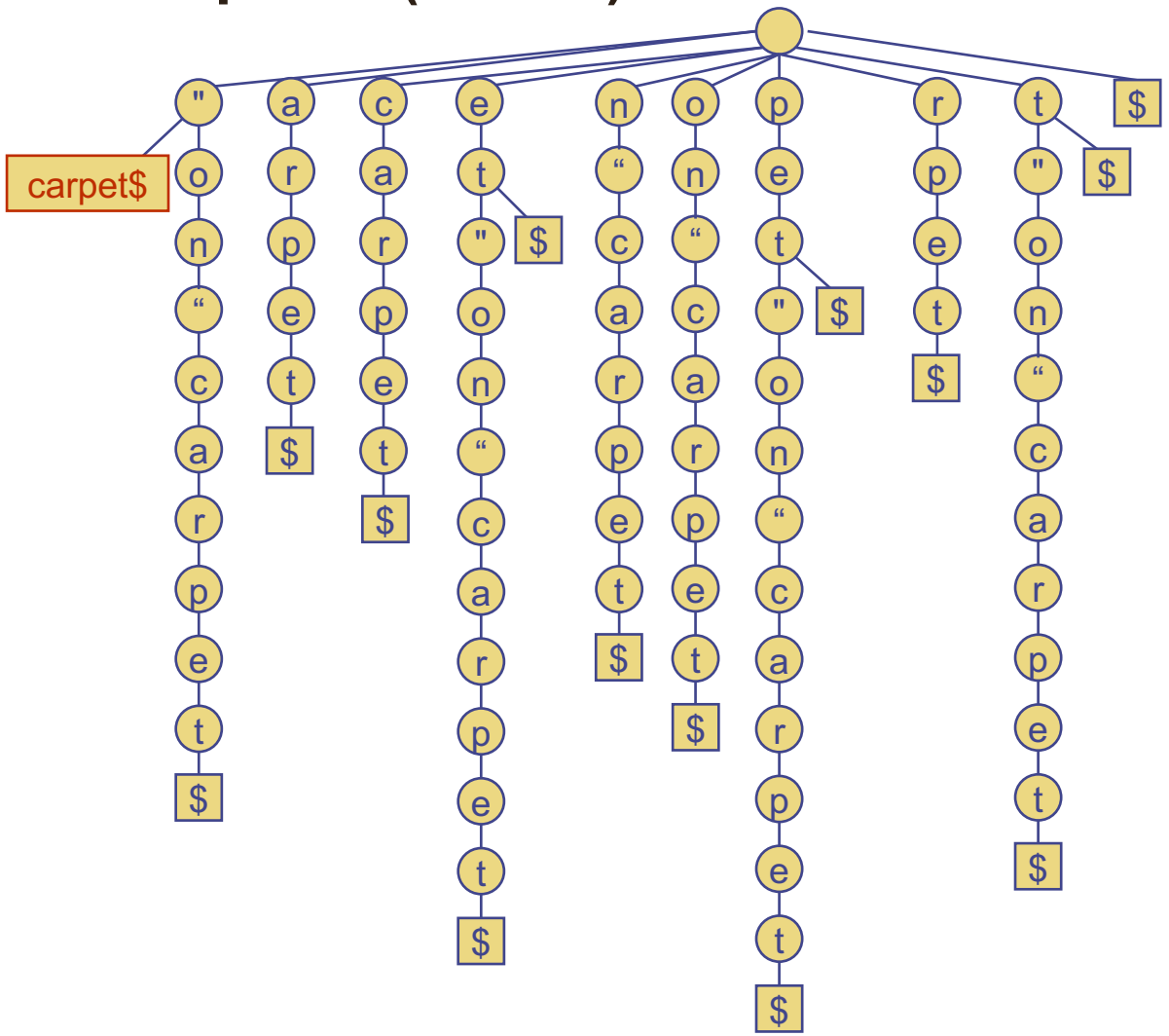
Suffix Trie Construction



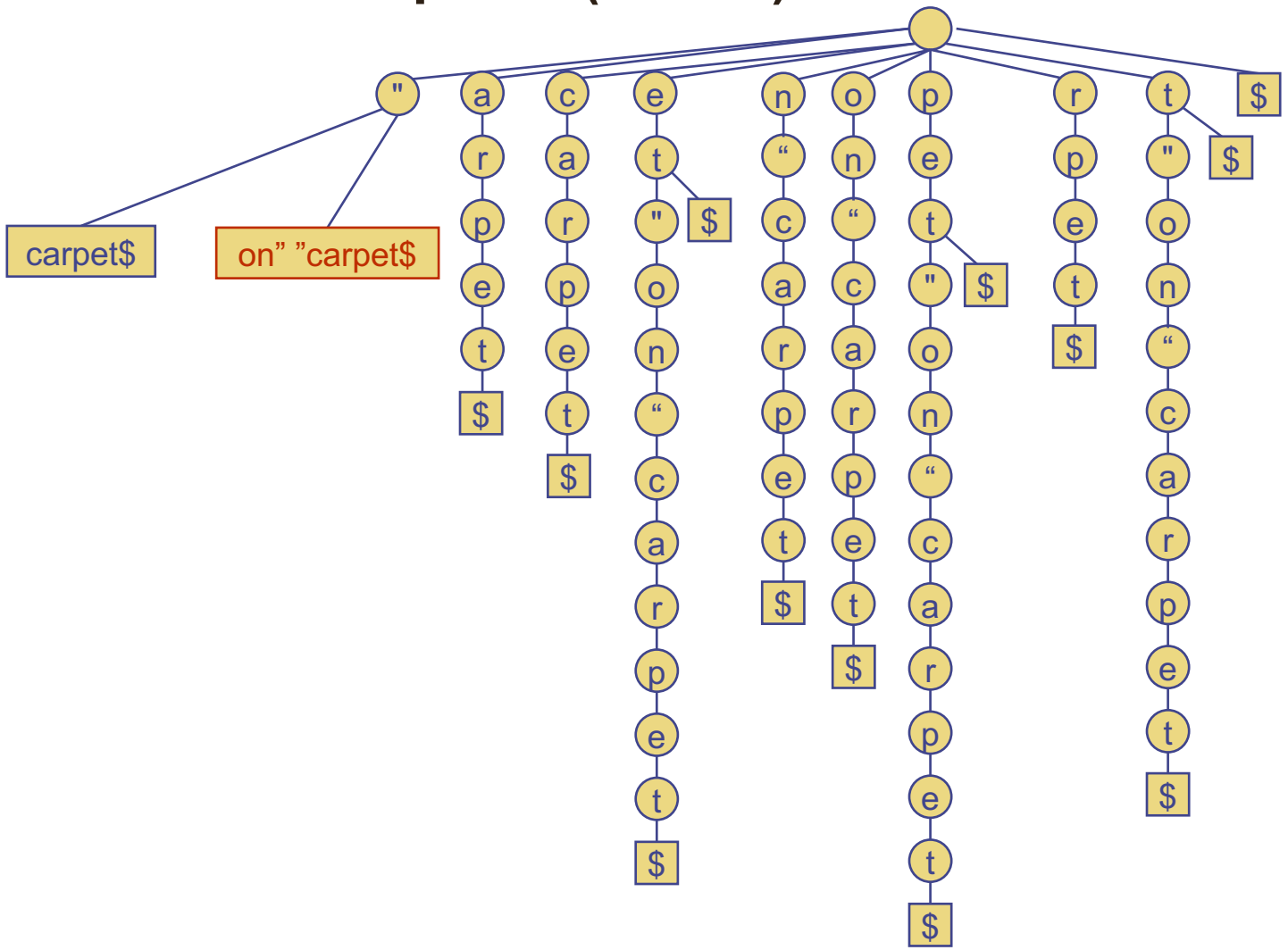
Suffix Trie: Compress



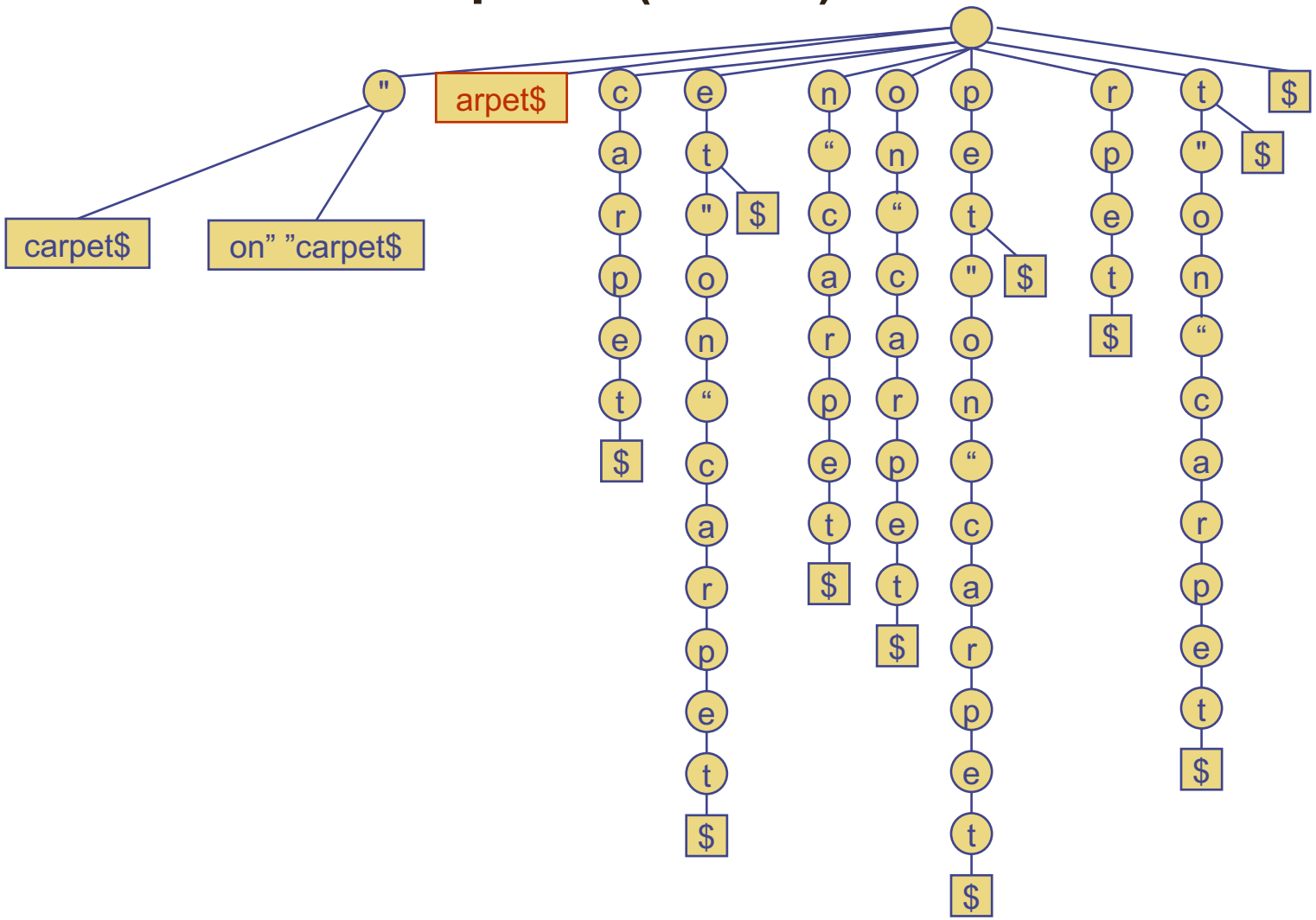
Suffix Trie: Compress (cont'd)



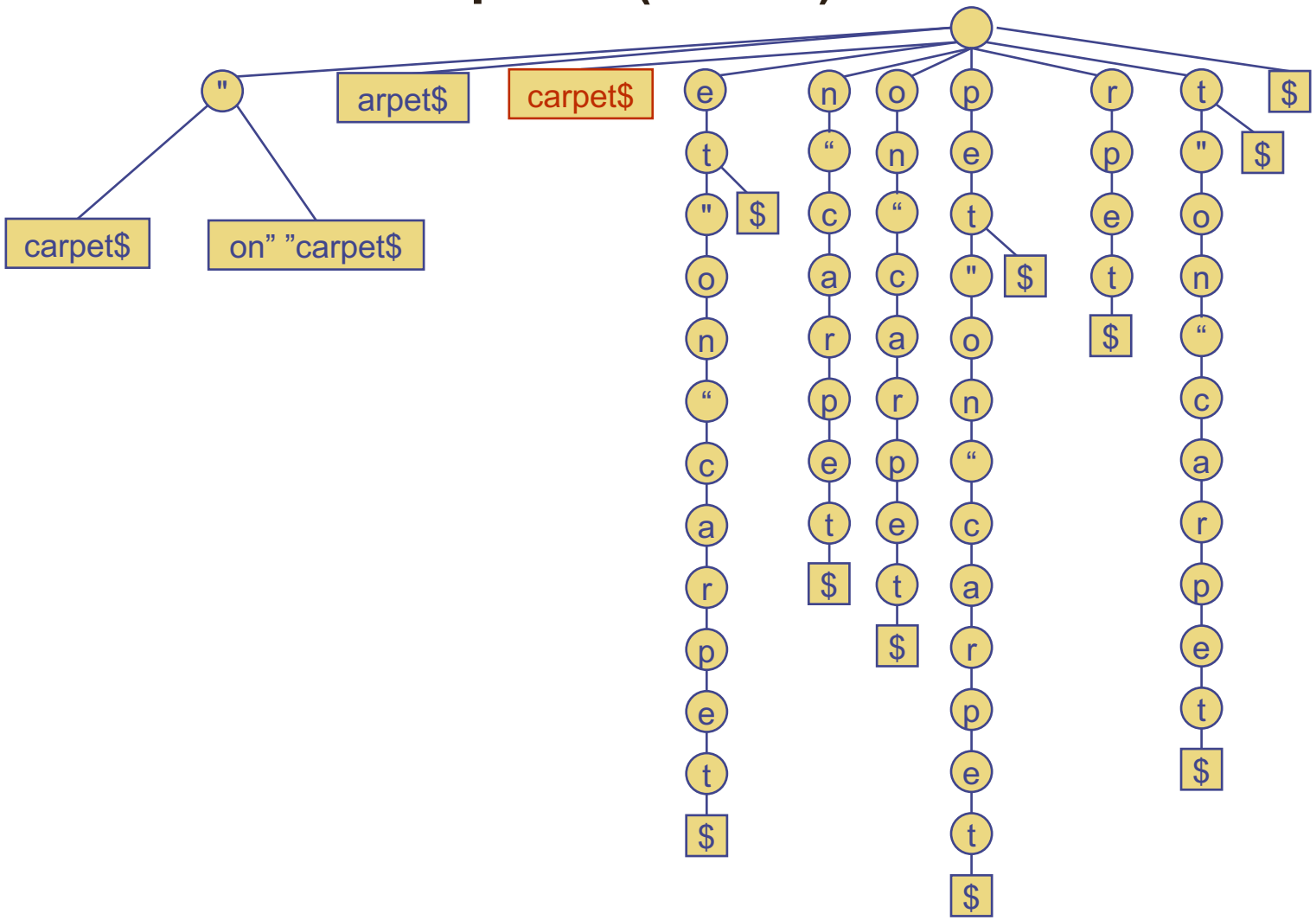
Suffix Trie: Compress (cont'd)



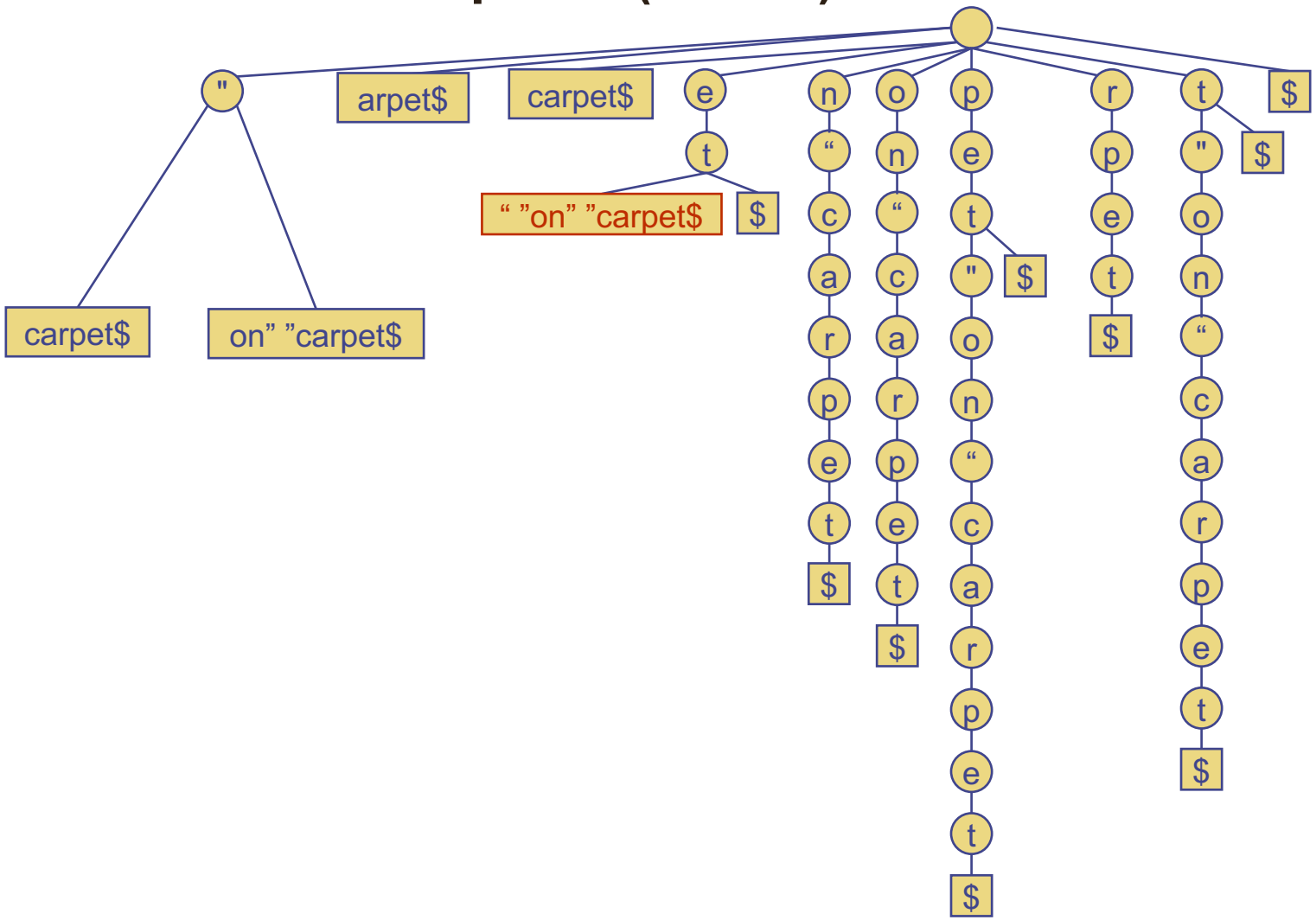
Suffix Trie: Compress (cont'd)



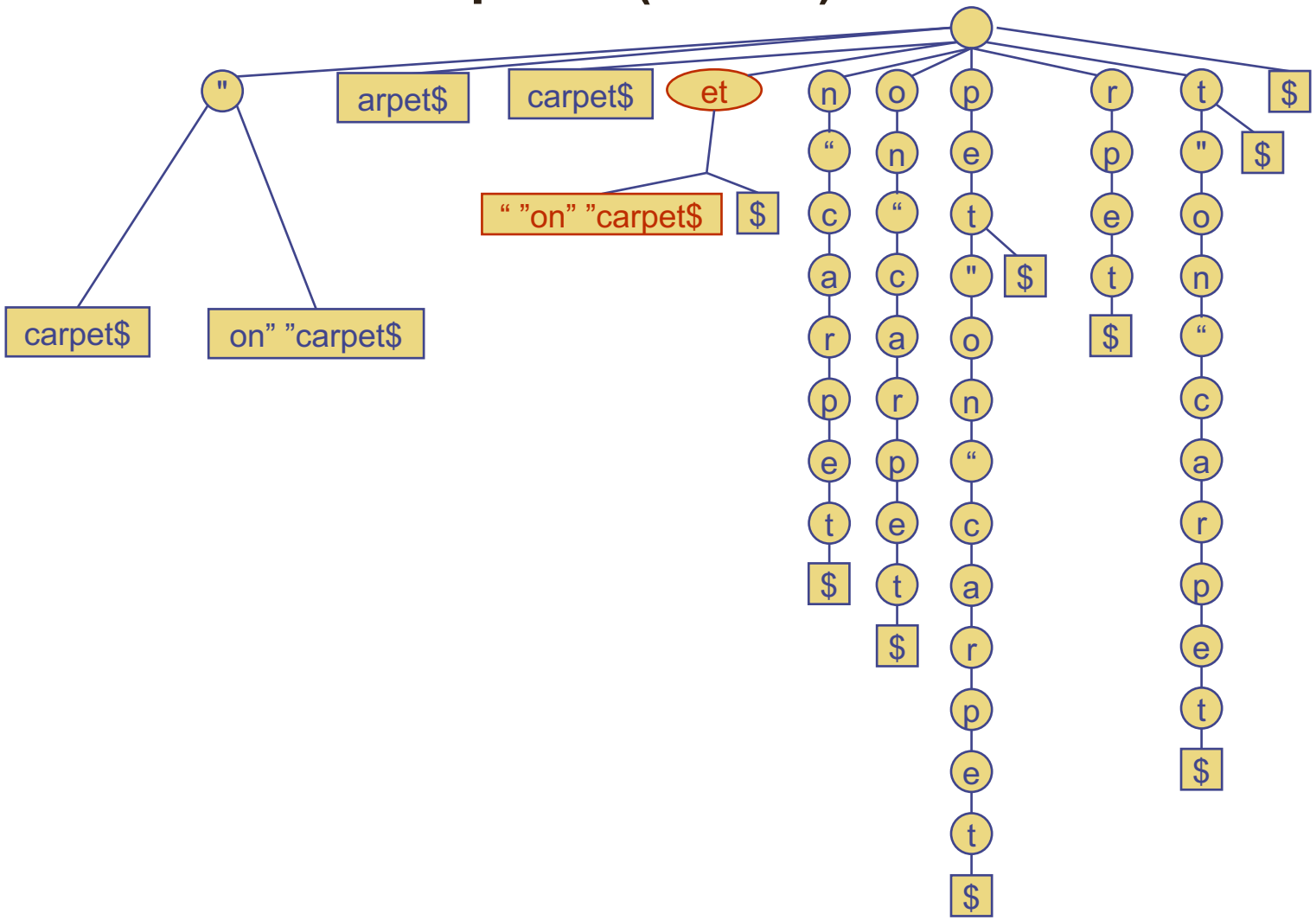
Suffix Trie: Compress (cont'd)



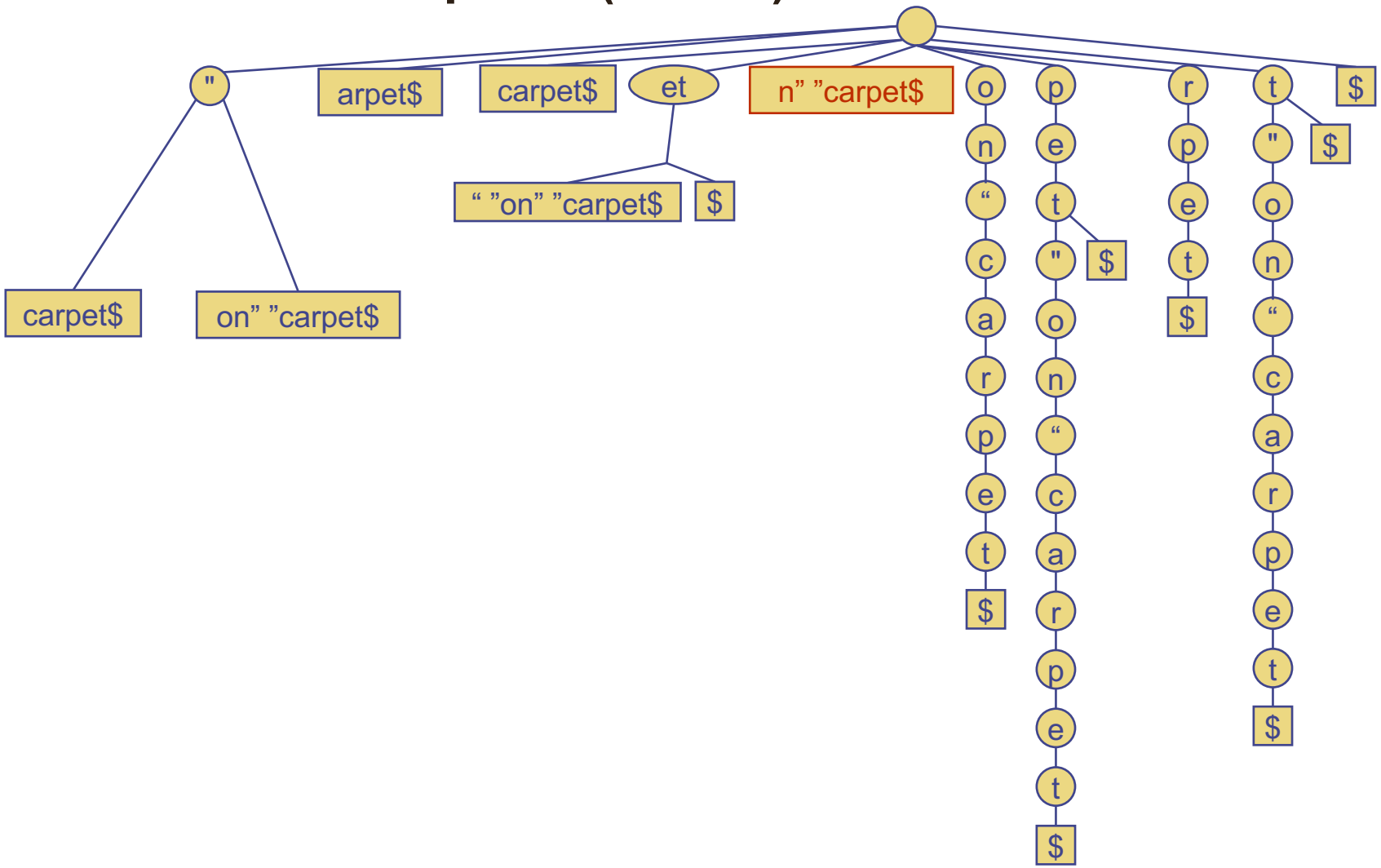
Suffix Trie: Compress (cont'd)



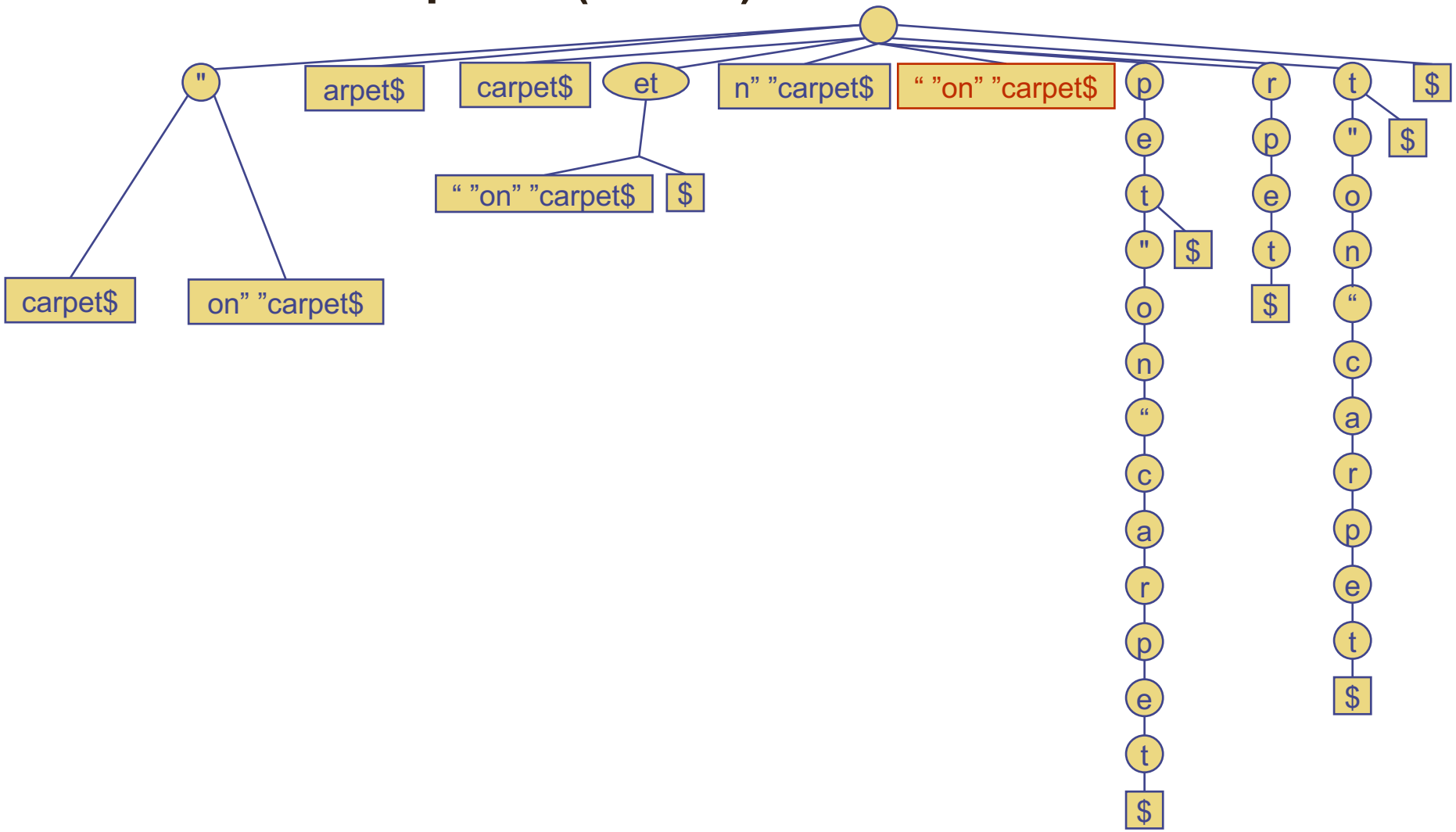
Suffix Trie: Compress (cont'd)



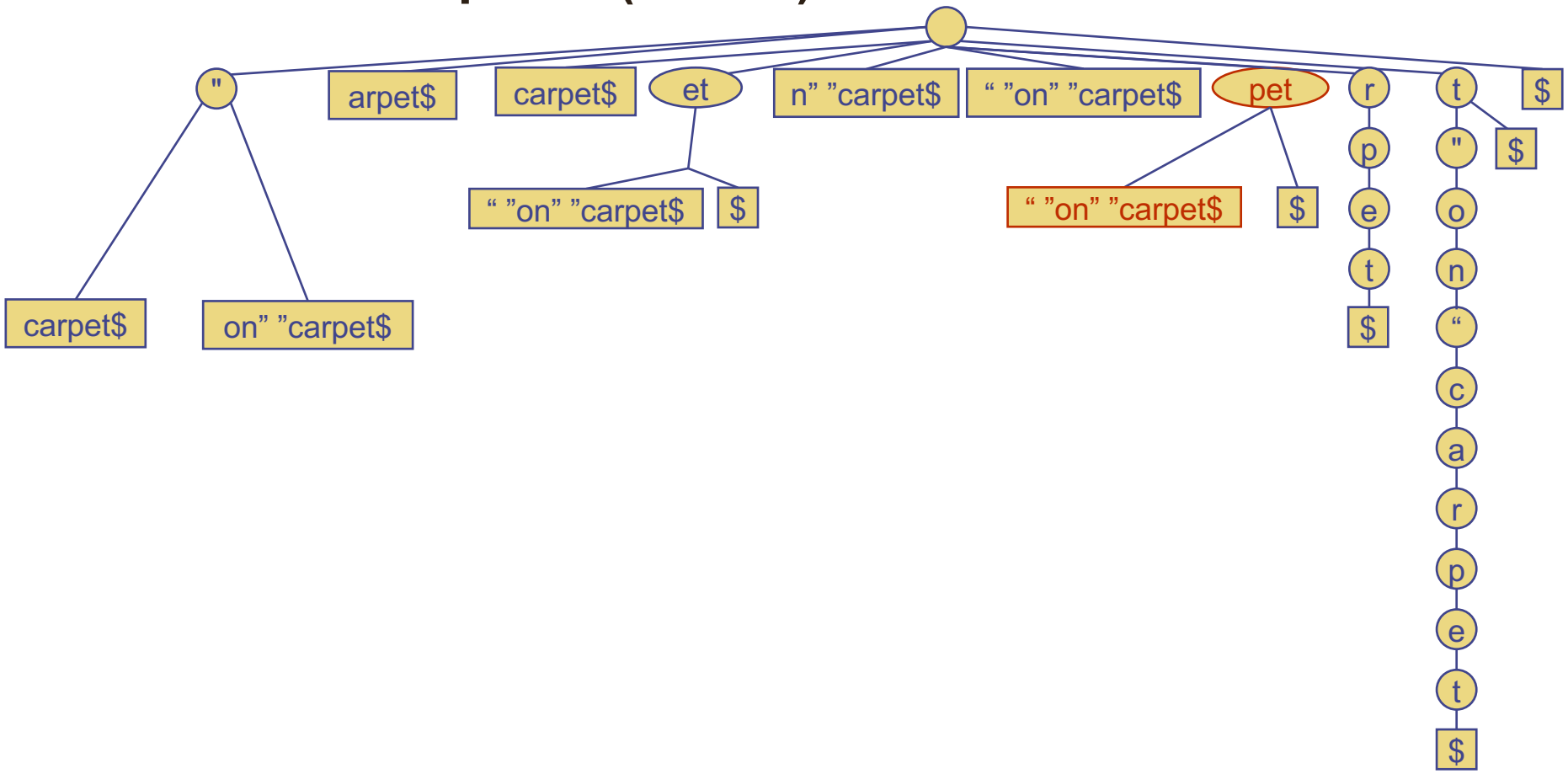
Suffix Trie: Compress (cont'd)



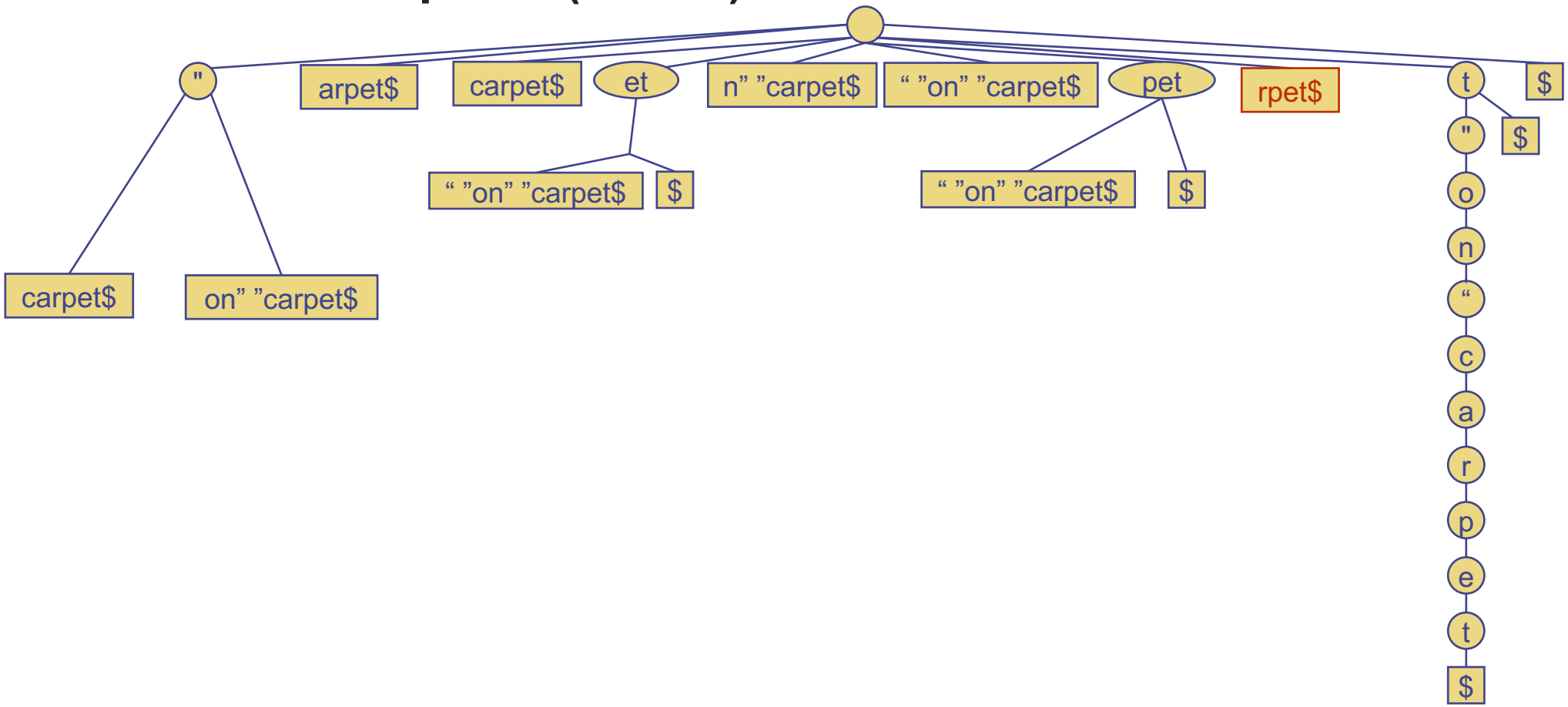
Suffix Trie: Compress (cont'd)



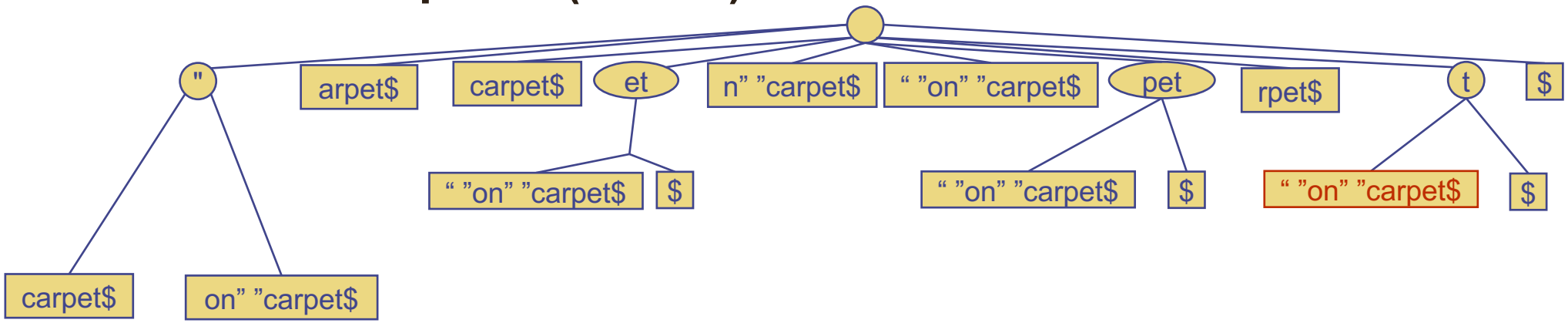
Suffix Trie: Compress (cont'd)



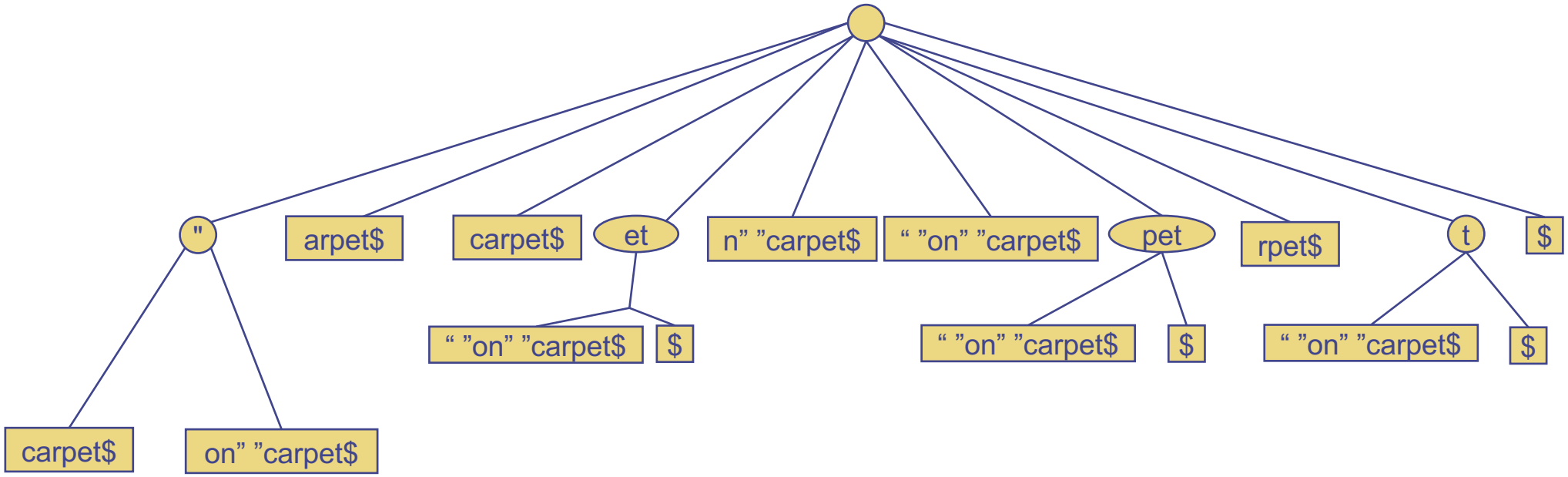
Suffix Trie: Compress (cont'd)



Suffix Trie: Compress (cont'd)

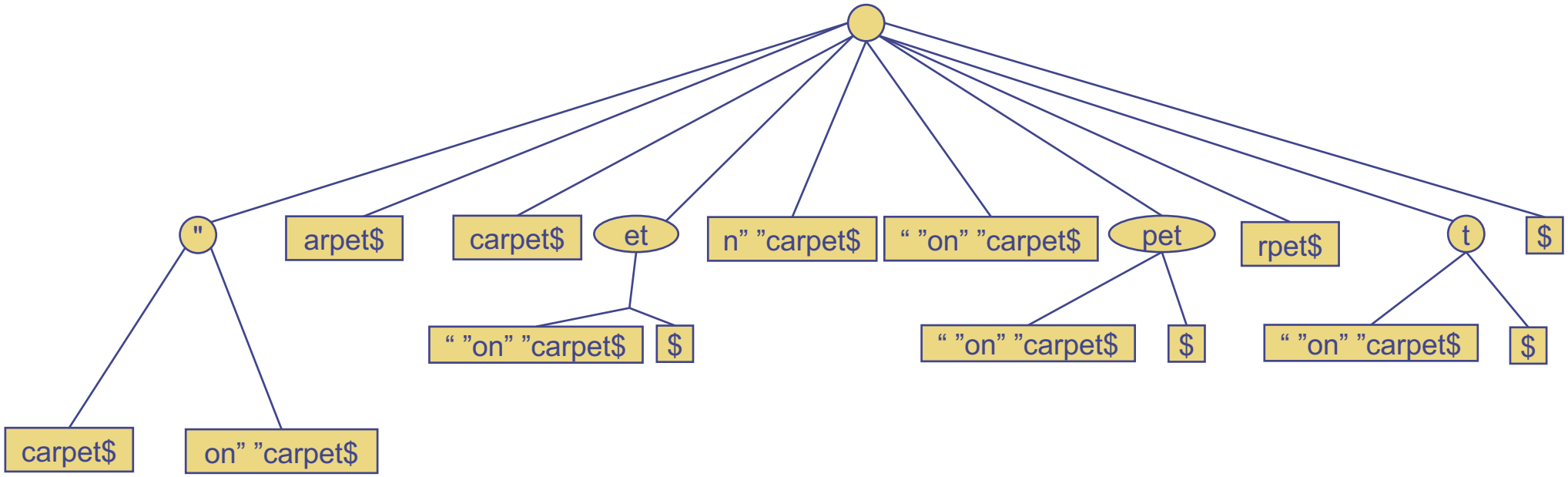


Suffix Trie: Index



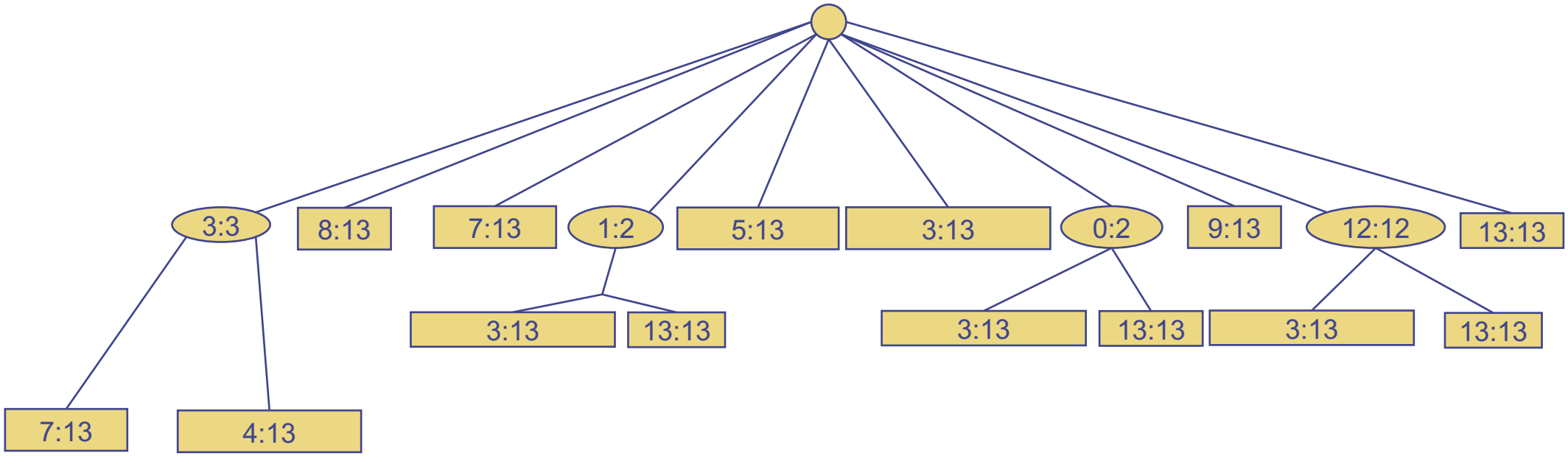
- Many substrings appear over and over again – impractical to store each label directly
- Instead, store only the indices in an array

Suffix Trie: Index (cont'd)



p	e	t	' '	o	n	' '	c	a	r	p	e	t	$
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Suffix Trie: Index (cont'd)



p	e	t	‘ ’	o	n	‘ ’	c	a	r	p	e	t	$
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Suffix Tries: Properties

- A suffix trie saves space compared to a standard trie, because of using compression
- The length of all the suffixes of a string X of length n is

$$n + (n-1) + (n-2) + \ \ldots + 2 + 1 = \frac{n(n+1)}{2}$$

- A standard trie uses $O(n^2)$ space to store all the suffixes of X
- A suffix trie (which is compressed) uses only $O(n)$ space
- A suffix trie can be constructed using the simple procedure described in $O(|\Sigma|n^2)$ time because the total length of the suffixes is quadratic in n
- For production use, there is are linear-time algorithms - $O(n)$ - for constructing suffix tries
  - P. Weiner. 1973. *Linear Pattern Matching Algorithms*.
  - E. Ukkonen. 1995. *On-line Construction of Suffix Trees*.

Thank you.